Growth

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1.05 Using The Tools

Give It a Try G2

DD: 1/14/13: Still one to finish. See comment..

DD: 1/13/13: One to finish.

DD: 1/11/13: A couple to reconsider.

Graphics Primitives

This LiveMath Independence Declaration allows the derivatives to be computed as intended in this notebook.

The variables $(a, b, c, x, y, t, r, k, s, z)$ are independent of [each other ▾].

G.2) Highest and lowest points on the graph

G.2.a)

Find the highest point on the graph of

$$f(x) = e^{-x^2}\left(2 + \cos(x) + \frac{\sin(x)}{2}\right).$$

Is there a lowest point on the graph?

$$y = e^{-x^2}\left(2 + \cos[x] + \frac{\sin[x]}{2}\right)$$

MR, 12/19: First I graph the function to ballpark the highest point.

$-1.5 \ldots 2.2$ = left...right [Stretch to Fit ▾]

$-1.1 \ldots 3.4$ = bottom...top cropping [Moderately ▾]

Graph Building Blocks

Curve at (x, y) where x = left ... right with a [normal ▾] line, colored [Blue ▾].

MR, 12/19: It looks like there 's one crest between -5 and 5.

$f(\mathfrak{X}) = e^{-\mathfrak{X}^2}\left(2+\cos[\mathfrak{X}]+\frac{\sin[\mathfrak{X}]}{2}\right)$

$f'(x) = \frac{d}{dx} f(x)$

$f'(x) = \left(\frac{1}{2}\cos[x] - \sin[x]\right) e^{-x^2} - 2\left(\cos[x] + \frac{1}{2}\sin[x] + 2\right) e^{-x^2} x$ *Substitute*

$f'(x) = \frac{1}{2} e^{-x^2} \cos(x) - 2 e^{-x^2} x \cos(x) - e^{-x^2} \sin(x) - e^{-x^2} x \sin(x) - 4 e^{-x^2} x$ *Expand*

$f'(\mathfrak{X}) = \frac{1}{2} e^{-\mathfrak{X}^2} \cos(\mathfrak{X}) - 2 e^{-\mathfrak{X}^2} \mathfrak{X} \cos(\mathfrak{X}) - e^{-\mathfrak{X}^2} \sin(\mathfrak{X}) - e^{-\mathfrak{X}^2} \mathfrak{X} \sin(\mathfrak{X}) - 4 e^{-\mathfrak{X}^2} \mathfrak{X}$

FindRootForFPrime $(-5,5)$

FindRootForFPrime $(-5,5) = 0.0705978476095161$ *Calculate*

MR, 12/19: This tells me the value for x where the original function is at its greatest. I plug in to solve:

$x = 0.070597847609516$

$f(x) = e^{-x^2}\left(2+\cos[x]+\frac{\sin[x]}{2}\right)$

$f(x) = \frac{\cos(0.070597847609516) + \frac{1}{2}\sin(0.070597847609516) + 2}{e^{0.00498405608709644}}$ *Substitute*

$f(x) = 3.01770068308235$ *Calculate*

MR, 12/19: Nailed it.

MR, 12/19: There is not a lowest point on the graph. The infinite limit in both positive and negative directions approaches 0 without ever reachign it because the dominant term is e^{-x^2}.

DD: 1/11/13: Good.

G.2.b)

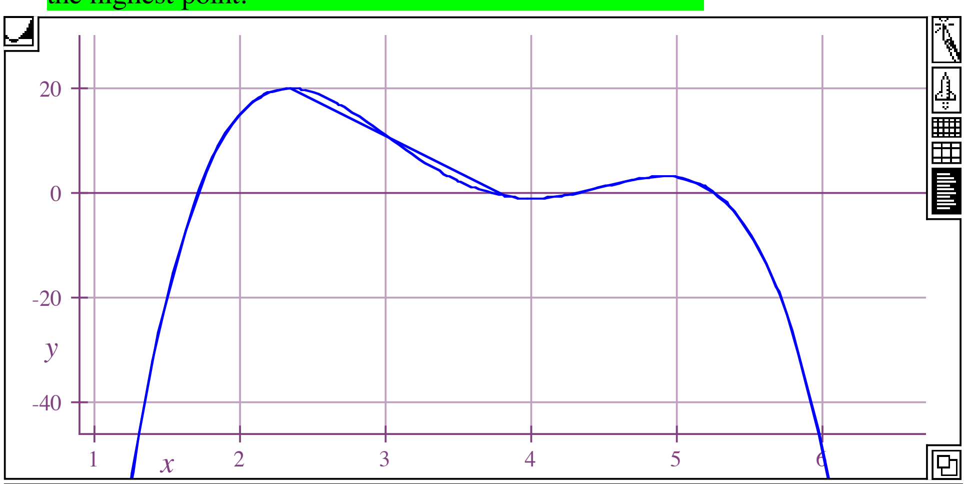
Find the highest point on the graph of

$f(x) = -577 + 736\,x - 324\,x^2 + 60\,x^3 - 4\,x^4$.

Is there a lowest point on this graph?

$y = -577 + 736\,x - 324\,x^2 + 60\,x^3 - 4\,x^4$

MR, 12/19: First I graph the function to ballpark the highest point.



0.9 ... 6.7 = left...right Stretch to Fit

-46 ... 30 = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored Blue.

MR, 12/19: It looks like the highest point is between 1 and 4.

$f(x) = -577 + 736x - 324x^2 + 60x^3 - 4x^4$

$f'(x) = \frac{d}{dx} f(x)$

$f'(x) = -16x^3 + 180x^2 - 648x + 736$ *Substitute*

$f'(x) = -16x^3 + 180x^2 - 648x + 736$ *Expand*

$f'(x) = -16x^3 + 180x^2 - 648x + 736$

FindRootForFPrime(1,4)

FindRootForFPrime(1,4) = 2.34413115425505 *Calculate*

MR, 12/19: This tells me the value for x where the original function is at its greatest. I plug in to solve:

$x = 2.344131154255$

$f(x) = -577 + 736x - 324x^2 + 60x^3 - 4x^4$

$f(x) = 19.9916393002012$ *Substitute*

MR, 12/19: Nailed it.

MR, 12/19: The lowest point is infinitely low. Since -4x is the dominant term, it keeps going lower and lower with no dip.

DD: 1/11/13: You don 't mean -4x is the dominant term. Please fix..

MR, 1/11: The lowest point is infinitely low. Since $-4x^4$ is the dominant term, it keeps going lower and lower with no dip.

DD: 1/13/13: Good.

G.2.c)

Find as accurately as you can the highest and lowest points on the graph of

$$f(x) = x\,\frac{240 - 7x^2}{240 + 3x^2}$$

for $-6 \le x \le 6$.

*******MR, 1/15: Here, with a factored polynomial: *******

$-21x^4 - 5760x^2 + 57600 = 0$

$-21(x + 3.10802202357547)(x - 3.10802202357547)(x - 16.8506829293279\,i)(x + 16.8506829293279\,i) = 0$ *Fac*

$x + 3.10802202357547 = 0$ *Isolate Each Monomial Factor*

$x = -3.10802202357547$ *Isolate*

$x - 3.10802202357547 = 0$ *Isolate Each Monomial Factor*

$x = 3.10802202357547$ *Isolate*

$x - 16.8506829293279\,i = 0$ *Isolate Each Monomial Factor*

$x = 16.8506829293279\,i$ *Isolate*

$x + 16.8506829293279\,i = 0$ *Isolate Each Monomial Factor*

$x = -16.8506829293279\,i$ *Isolate*

MR, 1/15: I look at only the real numbers:

$x = 3.10802202357547$

$x = -3.10802202357547$

MR, 1/15: Then I plug in to the orginal function to find the mins and maxes:

$x\frac{240-7x^2}{240+3x^2}$

MR, 1/15: This is the highest point:

$x\frac{240-7x^2}{240+3x^2} = 1.99184459004351$ *Substitute*

$f(3.1080220235755) = 1.9918445900435$

MR, 1/15: This is the lowest point:

$x\frac{240-7x^2}{240+3x^2} = -1.99184459004351$ *Substitute*

$f(-3.10802202357547) = -1.99184459004351$

MR, 1/15: Nice. Factoring gives me some actual x ' s to work with, rather than just a ballpark.

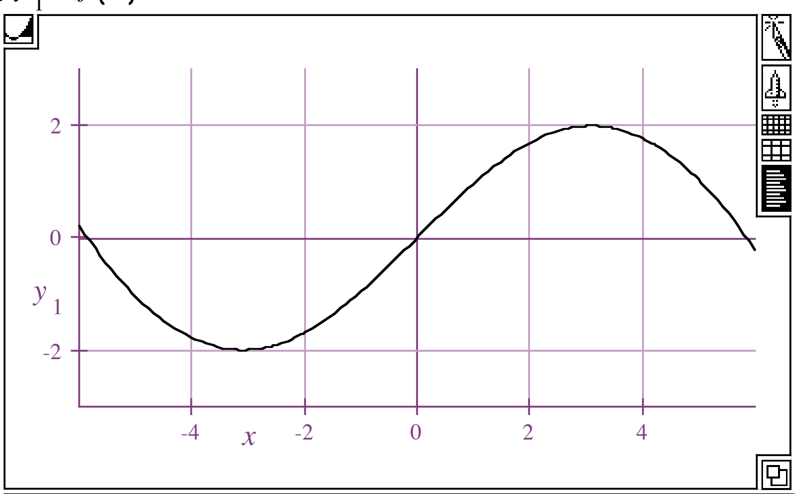
MR, 1/11: Fixed:

$y = x\frac{240-7x^2}{240+3x^2}$

MR, 1/11: It looks like the highest point is between x=2 and x=4. It looks like the lowest point is between x=-4 and -2.

$f(\mathfrak{x}) = \mathfrak{x}\frac{240-7\mathfrak{x}^2}{240+3\mathfrak{x}^2}$

$y_1 = f(x)$



$-6 \ldots 6$ = left...right True Proportions

$-3 \ldots 3$ = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y_1) where x = left ... right with a normal line, colored Black.

$f'(x) = \frac{d}{dx}f(x)$

$f'(x) = \frac{-6x^2(-7x^2+240)+(-21x^2+240)(3x^2+240)}{(3x^2+240)^2}$ *Substitute*

$$f'(x) = \frac{-21x^4 - 5760x^2 + 57600}{(3x^2+240)^2}$$ *Expand*

$$f'(x) = \frac{-21x^4 - 5760x^2 + 57600}{(3x^2+240)^2}$$

MR, 1/14: Now I set the function to 0 to solve find the roots:

$$0 = \frac{-21x^4 - 5760x^2 + 57600}{(3x^2+240)^2}$$

$$0 = -21x^4 - 5760x^2 + 57600$$ *Move Over*

$$0 = 3(-7x^4 - 1920x^2 + 19200)$$ *Collect*

$$0 = -7x^4 - 1920x^2 + 19200$$ *Move Over*

$$0 = -7\left(x + \frac{1}{2}\sqrt{\left[\sqrt{\left\{\frac{640}{7}\left(\left[\frac{1920}{7}\right]^2 + \frac{76800}{7}\right) - \frac{1}{27}\left(\frac{3840}{7}\right)^3\right\}^2 + \left\{\frac{1}{9}\left(3\left[\left\{\frac{1920}{7}\right\}^2 + \frac{76800}{7}\right] - \left[\frac{3840}{7}\right]^2\right)\right\}^3} + 1767370.26\right.}\right.$$

$$x = -\frac{1}{2}\sqrt{\left(\sqrt{\left[\frac{640}{7}\left\{\left(\frac{1920}{7}\right)^2 + \frac{76800}{7}\right\} - \frac{1}{27}\left\{\frac{3840}{7}\right\}^3\right]^2 + \left[\frac{1}{9}\left\{3\left(\left[\frac{1920}{7}\right]^2 + \frac{76800}{7}\right) - \left(\frac{3840}{7}\right)^2\right\}\right]^3} + 1767370.2623\right.}$$

$$x = -1.46000965999554 \times 10^{-7}\, i + \frac{1}{2}\sqrt{-\frac{1}{2}\left(\sqrt{\left[\frac{640}{7}\left\{\left(\frac{1920}{7}\right)^2 + \frac{76800}{7}\right\} - \frac{1}{27}\left\{\frac{3840}{7}\right\}^3\right]^2 + \left[\frac{1}{9}\left\{3\left(\left[\frac{1920}{7}\right]^2 + \right.\right.\right.}\right.}$$

MR, 1/14: I 'm having a very diffucult time isolating x. I 'm not sure I have the chops for this. Is this some kind of Pascal thing?

DD: 1/14/13: Easy. Write the polynomial with zero on the right hand side. Select the polynomial. Use the pull-down: Compute/Macros-Solving/ Solve-by-factoring. This will give you the solutions automatically. Do this neatly in a new, separate box. Things are getting disorganized here.

FindRootForFPrime $(-4,-2)$

FindRootForFPrime $(-4,-2) = -3.10802202357547$ *Calculate*

FindRootForFPrime $(2,4)$

FindRootForFPrime $(2,4) = 3.10802202357545$ *Calculate*

$x = -3.10802202357547$

$x = 3.1080220235755$

MR, 1/11: These tell me the values for x where the original function is at its least and greatest. I plug in to solve:

$$f(x) = x\,\frac{240 - 7x^2}{240 + 3x^2}$$

MR, 1/11: This is the lowest point:

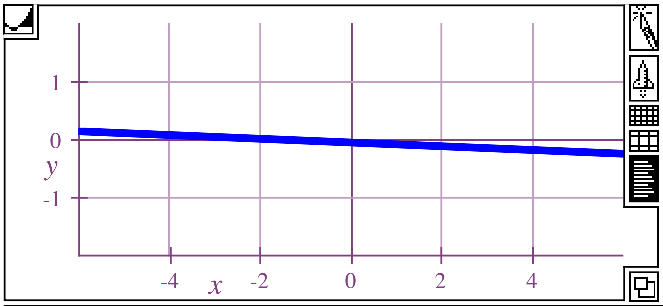
$f(x) = -1.99184459004351$ *Substitute*

MR, 1/11: This is the highest point:

$f(x) = 1.99184459004351$ *Substitute*

$$y = x\,\frac{240 - 7x^2}{240 + 3x^2}$$

MR, 12/19: First I graph the function to ballpark the highest point.



$-6 \ldots 6$ = left...right Stretch to Fit

$-2 \ldots 2$ = bottom...top cropping Moderately

Graph Building Blocks

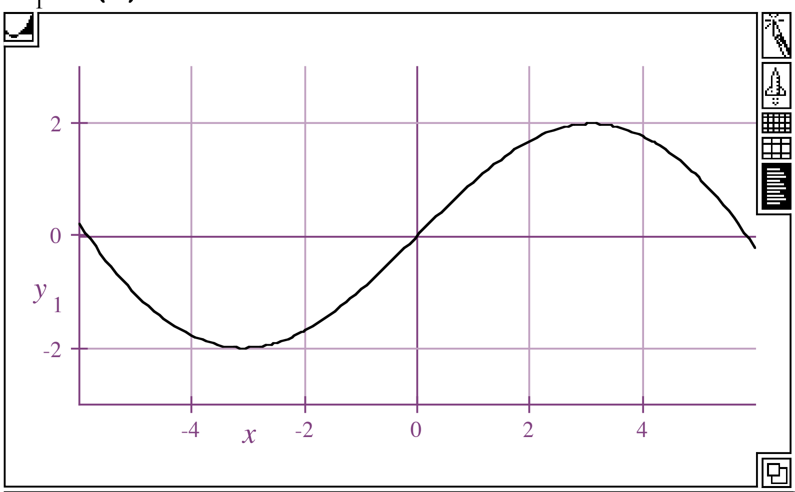
Curve at (x, y) where x = left ... right with a extra heavy line, colored Blue.

MR, 12/19: It looks like the highest point is at -6. The lowest point is at 6. This is a trap, since there is more stuff going on past the 6 's.

$$f(x) = x \frac{240 - 7x^2}{240 + 3x^2}$$

DD: 1/11/13: Here is a better graph of your function. You should reconsider your answers.

$$y_1 = f(x)$$



$-6 \ldots 6$ = left...right True Proportions

$-3 \ldots 3$ = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y_1) where x = left ... right with a normal line, colored Black.

$$f'(x) = \frac{d}{dx} f(x)$$

$$f'(x) = \frac{-6x^2(-7x^2 + 240) + (-21x^2 + 240)(3x^2 + 240)}{(3x^2 + 240)^2} \quad \textit{Substitute}$$

$$f'(x) = \frac{-21x^4 - 5760x^2 + 57600}{(3x^2 + 240)^2} \quad \textit{Expand}$$

$f'(x) = \frac{-21x^4 - 5760x^2 + 57600}{(3x^2 + 240)^2}$

DD: 1/13/13: I wouldn 't use FindRootForFPrime on this. You 'll be more accurate if you just solve the equation: f '(x) = 0.

$f'(x) = \frac{-21x^4 - 5760x^2 + 57600}{(3x^2 + 240)^2}$

$f'(x) = 0$

$\frac{-21x^4 - 5760x^2 + 57600}{(3x^2 + 240)^2} = 0$ *Substitute*

$-21x^4 - 5760x^2 + 57600 = 0$ *Move Over*

DD: 1/13/13: Set the numerator to zero and solve.

?

FindRootForFPrime (5.9999,6)

FindRootForFPrime (5.9999,6) = 5.9999 *Calculate*

FindRootForFPrime (−6,−5.9)

FindRootForFPrime (−6,−5.9) = −6 *Calculate*

$x = -6$

$x = 5.9999$

MR, 12/19: These tell me the values for x where the original function is at its least and greatest. I plug in to solve:

$f(x) = x\frac{240 - 7x^2}{240 + 3x^2}$

MR, 12/19: This is the highest point.

$f(x) = \frac{6}{29}$ *Substitute*

MR, 12/19: This is close to the lowest point, althought intuition tells me it should be where x=6. I 'm not sure why this didn 't calculate when I did the FindRootForFPrime fucntion.

$f(x) = -0.206750418262346$ *Substitute*

$x = 6$

$f(x) = -\frac{6}{29}$ *Substitute*

M5.3.b-2=8=011513.the

?