M1.3.b-2=3=012213.the

Growth

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1.01 Growth

Give It a Try G2

Graphics Primitives

G.2) Global scale*

G.2.a)

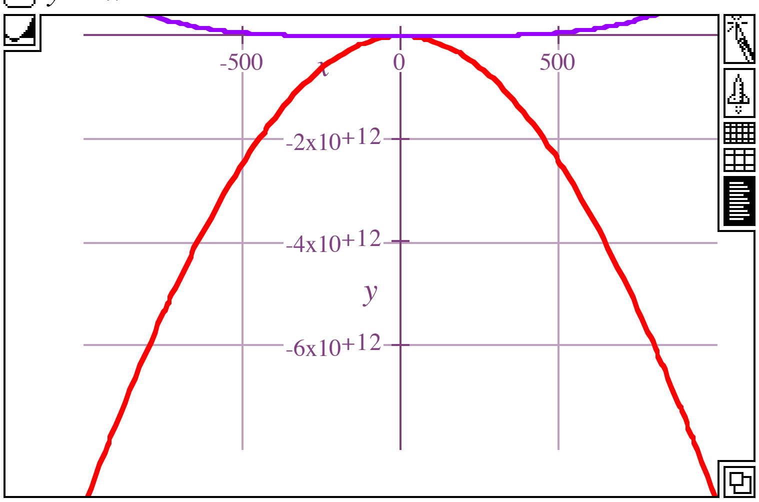
Look at:

$f(x) = x^4 - 10000000\,x^2$

N Ch 19.01.2013 A good global scale plote for this graph looks like f(x)=x4

RC: 01/21/13: Good

$y = x^4$



$-1000 \ldots 1000$ = left...right Stretch to Fit
$-8000000000000 \ldots 0$ = bottom...top cropping Moderately

Graph Building Blocks

Curve at $(x, f[x])$ where x = left ... right with a heavy line, colored Red.

Curve at (x, y) where x = left ... right with a heavy line, colored Purple.

Is this a good global scale plot of

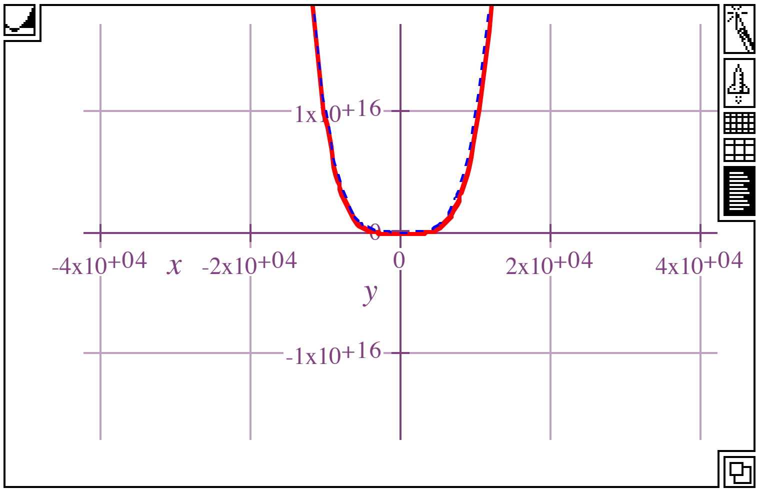
$$f(x) = x^4 - 10000000\,x^2 \; ?$$

Why or why not?

If it is not a good global scale plot of f(x), then give a good global scale plot of f(x).

$f(\mathfrak{x}) = \mathfrak{x}^4 - 10000000\,\mathfrak{x}^2$

$y = x^4$



– 42000 ... 42000 = left...right Stretch to Fit

– 1.7×10[16] ... 1.7×10[16] = bottom...top cropping Moderately

Graph Building Blocks

Curve at $(x, f[x])$ where x = left ... right with a heavy line, colored Red.

Curve at (x, y) where x = left ... right with a dashed line, colored Blue.

G.2.b)

Put

$$f(x) = \frac{2\,x^6 + 50\,x^2}{x^6 + 3\,x^2 + 1}.$$

What do you say are the limiting values

$$\lim_{x \to \infty} f(x)=2$$

and

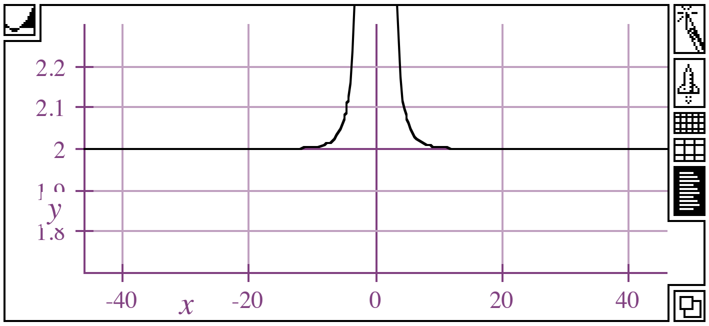
$$\lim_{x \to -\infty} f(x)=2$$

$$f(\mathfrak{x}) = \frac{2\,\mathfrak{x}^6 + 50\,\mathfrak{x}^2}{\mathfrak{x}^6 + 3\,\mathfrak{x}^2 + 1}$$

$$\frac{2\,x^6}{x^6}$$

$$\frac{2\,x^6}{x^6} = 2 \quad \textit{Simplify}$$

$y = f(x)$



$-46 \ldots 46$ = left...right   Stretch to Fit

$1.7 \ldots 2.3$ = bottom...top   cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored Black.

RC: 01/21/13: Your y-axis scale is too large to show y=2 as the limiting value. Your scale should be something like 1.5....2.5

N Ch 22.01.2013 Fixed it.

RC: 01/22/13: Good

G.2.c)

What do you say is the limiting value

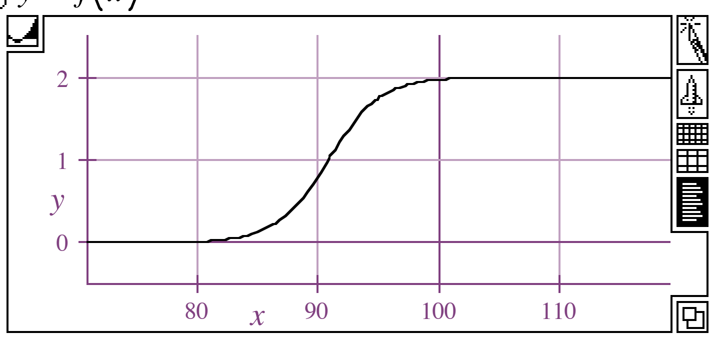
$$\lim_{x \to \infty} \frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}} = 2$$

Illustrate with a plot.

$$f(x) = \frac{x^9 + 4e^{0.6x}}{3x^{12} + 2e^{0.6x}}$$

RC: 01/22/13: good

$y = f(x)$



$71 \ldots 119$ = left...right   Stretch to Fit

$-0.5 \ldots 2.5$ = bottom...top   cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored Black.

N Ch 19.01.2013 "exponent part" doesn 't depend on x

RC: 01/21/13: Incorrect. Go out farther to the right on your graph. What do your computations below have to do with this graph?

$$\frac{4\,e^{0.6x}}{2\,e^{0.6x}}$$

$$\frac{4\,e^{0.6x}}{2\,e^{0.6x}} = 2 \quad \textit{Simplify}$$

RC: 01/22/13: You don 't need the "power part" since the exponentials dominate.

"Power part"

$$\frac{x^{9}}{3\,x^{12}}$$

$$\frac{x^{9}}{3\,x^{12}} = \frac{1}{3\,x^{3}} \quad \textit{Simplify}$$

$$\lim_{x \to \infty} \frac{1}{3\,x^{3}} = 0$$

N Ch 22.01.2013 Thank God! I was really uncomfortable with that zero limit. My problem is I 'm terrified of limits. I 'm still not sure what they are.

So, for small x, f(x) tends to 0

$$\frac{50^{9}+4\,e^{0.6\cdot 50}}{3\cdot 50^{12}+2\,e^{0.6\cdot 50}}$$

$$\frac{50^{9}+4\,e^{0.6\cdot 50}}{3\cdot 50^{12}+2\,e^{0.6\cdot 50}} = 2.72502898699495\times 10^{-6} \quad \textit{Calculate}$$

then, for large x, it goes up to2

$$\frac{100^{9}+4\,e^{0.6\cdot 100}}{3\cdot 100^{12}+2\,e^{0.6\cdot 100}}$$

$$\frac{100^{9}+4\,e^{0.6\cdot 100}}{3\cdot 100^{12}+2\,e^{0.6\cdot 100}} = 1.97407104287832 \quad \textit{Calculate}$$

Am I right?

G.2.d)

What do you say is the limiting value

$$\lim_{x \to \infty} \frac{3\,x^{8} - 123\,\cos(x) - 6\,x^{2}}{e^{0.4x}} = 0$$

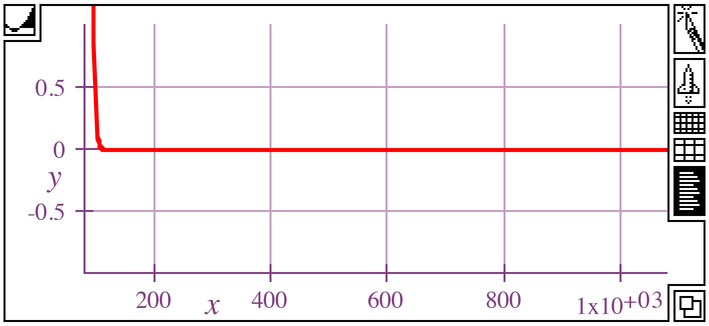
Illustrate with a plot.

$$f(x) = \frac{3\,x^{8} - 123\,\cos\left(\left|x\right|\right) - 6\,x^{2}}{e^{0.4x}}$$

RC: 01/21/13: Your y-axis scale is too large to show y=0 as the limiting value.

N Ch 22.01.2013 Fixed it.

$y = f(x)$



80 ... 1080 = left...right Stretch to Fit

− 1 ... 1 = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a heavy line, colored Red.

N Ch 19.01.2013 a limit here depends on an exponent in the denominator

RC: 01/22/13: Good

G.2.e)

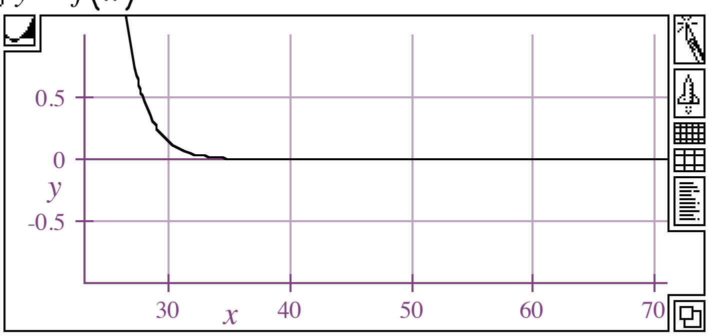
What do you say is the limiting value

$$\lim_{x \to \infty} e^{-0.8x}\left(1+5\,x^{6}\right) = 0$$

Illustrate with a plot.

$f(x) = e^{-0.8x}\left(1+5x^{6}\right)$

$y = f(x)$



N Ch 19.01.2013 $e^{-0.8x} = \frac{1}{e^{0.8x}}$

RC: 01/21/13: Your y-axis scale is too large to show y=0 as the limiting value.

RC: 01/22/13: Good

N Ch 22.01.2013 Fixed it.

G.2.f)

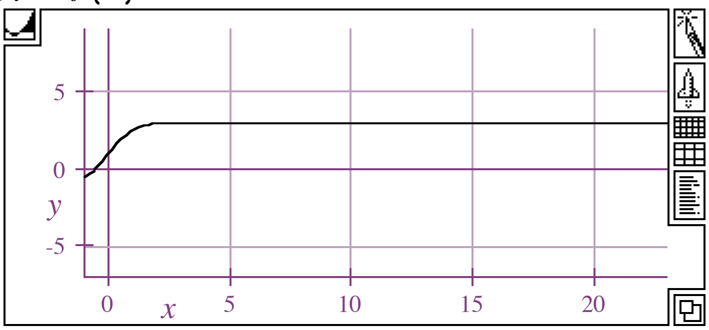
What do you say is the limiting value

$$\lim_{x \to \infty} \frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = 3$$

Illustrate with a plot.

$$f(x) = \frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}}$$

$$y = f(x)$$



$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{e^{3x}\left(3e^{-x} - e^{-3x}\right)}{e^{3x}\left(e^{-3x} + e^{-x}\right)} \quad \textit{Apply}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{-e^{\frac{1}{1}(-3x)+\frac{1}{1}(3x)} + 3e^{\frac{1}{1}(-x)+\frac{1}{1}(3x)}}{e^{3x}\left(e^{-3x} + e^{-x}\right)} \quad \textit{Expand}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{3e^{-x+3x} - e^{-3x+3x}}{e^{3x}\left(e^{-3x} + e^{-x}\right)} \quad \textit{Simplify}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{3e^{2x} - 1}{e^{3x}\left(e^{-3x} + e^{-x}\right)} \quad \textit{Simplify}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{3e^{2x} - 1}{e^{\frac{1}{1}(-x)+\frac{1}{1}(3x)} + e^{\frac{1}{1}(-3x)+\frac{1}{1}(3x)}} \quad \textit{Expand}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{3e^{2x} - 1}{e^{-x+3x} + e^{-3x+3x}} \quad \textit{Simplify}$$

$$\frac{3e^{-x} - e^{-3x}}{e^{-3x} + e^{-x}} = \frac{3e^{2x} - 1}{e^{2x} + 1} \quad \textit{Simplify}$$

$$\frac{3e^{2x}}{e^{2x}}$$

$$\frac{3e^{2x}}{e^{2x}} = 3 \quad \textit{Simplify}$$

RC: 01/21/13: Good

Tip
...

G.2.g)

Rank the following functions in order of dominance as $x \to \infty$:

$$0.0001\, x^{24},\ 0.0004\, e^{0.01x},\ 89\, x^2,\ \sqrt{x},\ 17\, x,\ 0.08\, x^3, 0.0000013\, e^{2x},\ 100\, x^{0.4}.$$

N Ch 19.01.2013 - $0.0000013\, e^{2x}, 0.0004\, e^{0.01x}$, $0.0001\, x^{24}, 0.08\, x^3, 89\, x^2, 17\, x, \sqrt{x}, 100\, x^{0.4}$

RC: 01/21/13: Good

G.2.h)

Plot

$$f(x) = \frac{2\,x^4 - 40\,x + 1}{x^2 + x + 12}$$

in global scale.

What simpler function mimicks the global scale behavior of $f(x)$?

Give a number b so that $f(x)$ is in its global scale behavior for $|x| > b$.

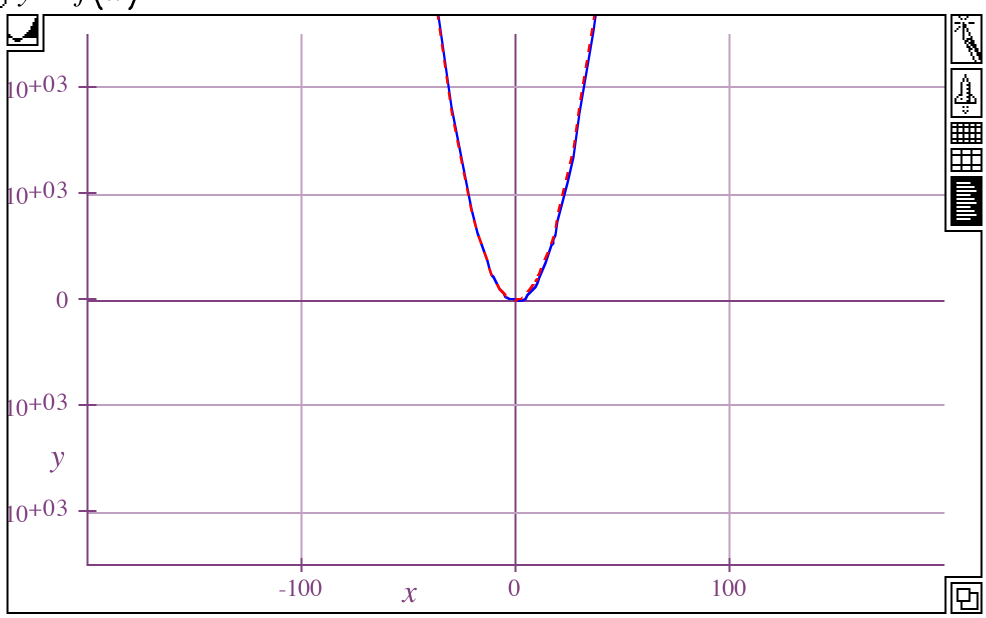
$$f(x) = \frac{2x^4 - 40x + 1}{x^2 + x + 12}$$

RC: 01/21/13: b=2 is probably sufficient.

N Ch 22.01.2013 may I choose b=8 ? Or b=2 is enough?

RC: 01/22/13: You may choose any number past b=1 or so. The problem is a little vague (on purpose)

$y = f(x)$



$-200 \ldots 200$ = left...right Stretch to Fit

$-2500 \ldots 2500$ = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored Blue.

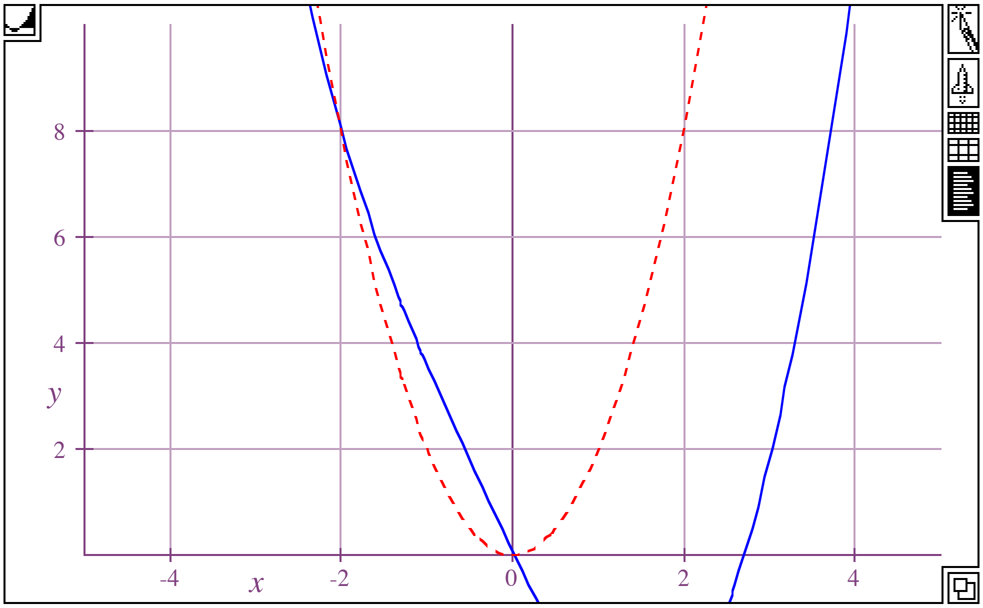
Blue.

Curve at (x, y_1) where x = left ... right with a dashed line, colored

Red.

$y_1 = 2x^2$

$b = 8$



$-5 \ldots 5$ = left...right Stretch to Fit

$0 \ldots 10$ = bottom...top cropping Moderately

Graph Building Blocks

Curve at (x, y) where x = left ... right with a normal line, colored

Blue.

Curve at (x, y_1) where x = left ... right with a dashed line, colored

Red.

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