Growth

Authors : Bill Davis, Horacio Porta and Jerry Uhl **Producer** : Bruce Carpenter
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1.03 Instantaneous Growth Rates

Give It a Try G6

DD: 1/9/13: Good. Notebook is complete.

DD: 1/6/13: Still a few to fix.

DD: 1/1/13: Still more to do.

DD: 12/28/12: A few to reconsider.

Experience with the starred problems will be especially beneficial for understanding later lessons.

Graphics Primitives

G.6) Up and down, maximum and minimum*

G.6.a)

You can tell what happens to

$$f(x) = x^3 - 3x^2$$

as x leaves $x = 2.6$ and advances a little bit by $f'(2.6)$:

$f(x) = x^3 - 3x^2$

$f'(x) = \left[\frac{d}{dx} f(x)\right]_{x=x}$

$f'(x) = \left[\frac{d}{dx}(x^3 - 3x^2)\right]_{x=x}$ *Substitute*

$f'(x) = \left[3x^2 - 6x\right]_{x=x}$ *Simplify*

$f'(x) = 3x^2 - 6x$ *Simplify*

$f'(2.6)$

$f'(2.6) = 3 \cdot 2.6^2 - 6 \cdot 2.6$ *Substitute*

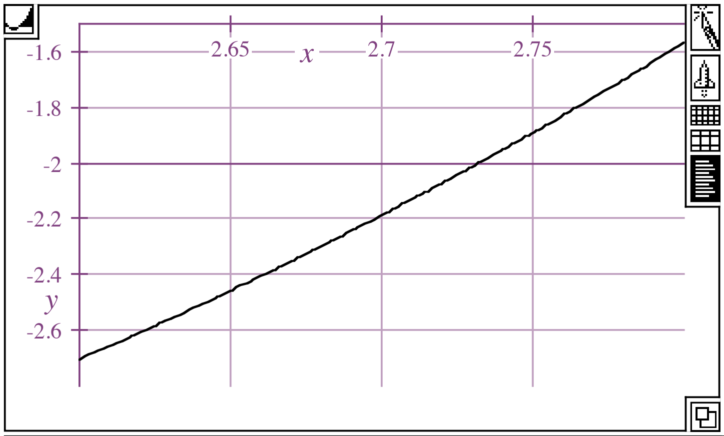
$f'(2.6) = 4.68$ *Simplify*

Positive.

This means $f(x)$ increases as x leaves 2.6 and advances a little bit.

Check with a plot:

$f(x) = x^3 - 3x^2$



2.6 ... 2.8 = left...right Stretch to Fit

– 2.8 ... – 1.5 = bottom...top cropping Moderately

Graph Building Blocks

Curve at $(x, f[x])$ where x = left ... right with a normal line, colored Black.

Yep.

As x leaves 2.6 and advances a little bit, f(x) goes up.

Stay with $f(x) = x^3 - 3x^2$ and look at:

$f(\mathfrak{x}) = \mathfrak{x}^3 - 3\mathfrak{x}^2$

$f'(\mathfrak{x}) = {}_{x=\mathfrak{x}}\left[\frac{d}{dx} f(x)\right.$

$f'(\mathfrak{x}) = {}_{x=\mathfrak{x}}\left[\frac{d}{dx}(x^3 - 3x^2)\right.$ *Substitute*

$f'(\mathfrak{x}) = {}_{x=\mathfrak{x}}\left[3x^2 - 6x\right.$ *Simplify*

$f'(\mathfrak{x}) = 3\mathfrak{x}^2 - 6\mathfrak{x}$ *Simplify*

$f'(1.7)$

$f'(1.7) = 3 \cdot 1.7^2 - 6 \cdot 1.7$ *Substitute*

$f'(1.7) = -1.53$ *Simplify*

As x leaves x = 1.7 and advances a little bit, does

$f(x) = x^3 - 3x^2$

go up or down?

Confirm with a plot.

MR, 12/13: Since the result is negative, x will go down by a little bit as it leaves 1.7.

$f(\mathfrak{x}) = \mathfrak{x}^3 - 3\mathfrak{x}^2$

$f'(x) = \frac{d}{dx} f(x)$

$f'(x) = \frac{d}{dx}(x^3 - 3x^2)$ *Substitute*

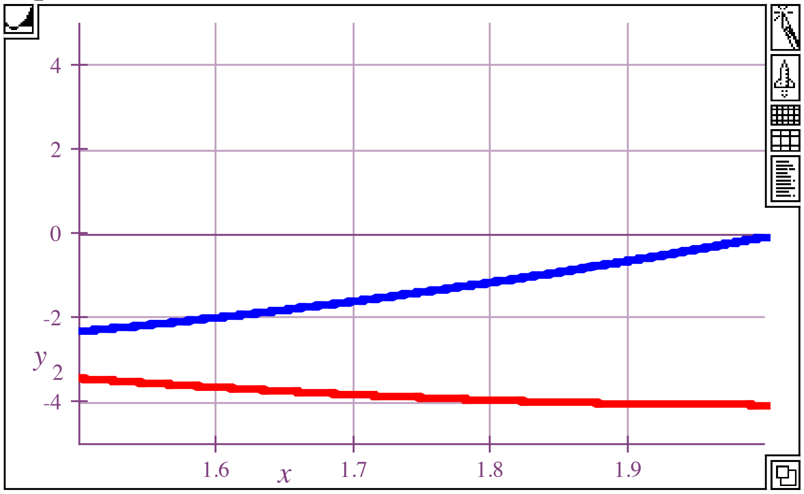
$f'(x) = -6x + 3x^2$ *Expand*

$f(\mathfrak{x}) = \mathfrak{x}^3 - 3\mathfrak{x}^2$

$f'(\mathfrak{x}) = -6\mathfrak{x} + 3\mathfrak{x}^2$

$y_1 = f(x)$

$y_2 = f'(x)$



MR, 12/13: Confirmed. f '(x) (blue) is negative and f(x) (red) is goind down.

DD: 12/28/12: Good.

G.6.b)

This time go with

$f(x) = x^4 - 4x^2$

and look at:

$f(\mathfrak{x}) = \mathfrak{x}^4 - 4\mathfrak{x}^2$

$f'(\mathfrak{x}) = \left[\frac{d}{dx} f(x)\right]_{x=\mathfrak{x}}$

$f'(\mathfrak{x}) = \left[\frac{d}{dx}(x^4 - 4x^2)\right]_{x=\mathfrak{x}}$ *Substitute*

$f'(\mathfrak{x}) = \left[4x^3 - 8x\right]_{x=\mathfrak{x}}$ *Simplify*

$f'(\mathfrak{x}) = 4\mathfrak{x}^3 - 8\mathfrak{x}$ *Simplify*

$f'(1.3)$

$f'(1.3) = 4 \cdot 1.3^3 - 8 \cdot 1.3$ *Substitute*

$f'(1.3) = -1.612$ *Simplify*

What happens to $f(x)$ as x leaves $x = 1.3$ and increases a little bit?

What happens to $f(x)$ as x leaves 1.3 and decreases a little bit?

What happens to $f(x)$ as x leaves 2.6 and increases a little bit?

What happens to $f(x)$ as x leaves 2.6 and decreases a little bit?

MR, 12/13. OK, I 'll solve algrebraically.

$f(\mathfrak{x}) = \mathfrak{x}^4 - 4\mathfrak{x}^2$

$f'(\mathfrak{x}) = \left[\frac{d}{dx} f(x)\right]_{x=\mathfrak{x}}$

$f'(\mathfrak{x}) = \left[\frac{d}{dx}(x^4 - 4x^2)\right]_{x=\mathfrak{x}}$ *Substitute*

$f'(\mathfrak{x}) = \left[4x^3 - 8x\right]_{x=\mathfrak{x}}$ *Simplify*

$f'(x) = 4x^3 - 8x$ *Simplify*

$f'(2.6)$

$f'(2.6) = 4 \cdot 2.6^3 - 8 \cdot 2.6$ *Substitute*

$f'(2.6) = 49.504$ *Simplify*

?

What happens to $f(x)$ as x leaves $x = 1.3$ and increases a little bit?

MR, 12/13: Since f '(1.3) is negative, f(x) will decrease leaving x=1.3.

What happens to $f(x)$ as x leaves 1.3 and decreases a little bit?

MR, 12/13: The opposite of the above case is true. Since f '(1.3) is negative, when f(x) moves to the left of 1.3, it will increase.

What happens to $f(x)$ as x leaves 2.6 and increases a little bit?

MR, 12/13: At f '(x), 2.6=49.504. Since f '(2.6) is positive, f(x) will increase leaving x=2.6.

What happens to $f(x)$ as x leaves 2.6 and decreases a little bit?

MR, 12/13: The opposite of the above case is true. Since f '(2.6) is positive, when f(x) moves to the left of 2.6, it will decrease.

DD: 12/28/12: Good.

G.6.c.i)

You 've got a function $f(x)$ and a point $x = a$.

If $f'(a)>0$, is it possible that $f(a) \le f(x)$ for all other x 's?

Why?

MR, 1/6: Since f '(a) is positive, when x moves to the right of a, then f(x) will increase and when x moves to the left of a, then f(x) will decrease. It is not possible for $f(a) \le f(x)$ for all other x 's. If this were the case, f(x) would be at its global minimum and f '(a) would = 0.

DD: 1/9/13: Good.

MR, 1/2: Thanks for the video. Since f '(a) is positive, when f(x) moves to the right it will increase and when it moves to the left it will decrease. It is not possible for $f(a) \le f(x)$ for all other x 's. If this were the case, f(x) would be at its global minimum and f '(a) would = 0.

DD: 1/6/13: You don 't mean "f(x) moves to the right." What you mean is when x moves to the right of a, then f(x) will increase. Etc. Please change this one and the others below.

MR, 12/13: Genearlly, the answer to each of these questions is yes since x could be an infinite number of inputs.

MR, 12/13. To wrap my mind around this, I 'll put it in more concrete terms. Imagine that f '(1)=1. This indicates that at f(1), f(x) is increasing, but moving to the left, f(x) is decreasing. So, yes, it is possible for f(a) to be less than or equal to f(x).

DD: 12/28/12: Read this more carefully. Basically, you 're being asked this: if f '(a) > 0 is it possible that f(a) is the minimum value of f(x). That would be the case if f(a) ≤ f(x) for all other x 's. Discussing a specific example is not a bad thing, but in the end you must answer the question is all its generality and give a reasonable argument to support your answer.

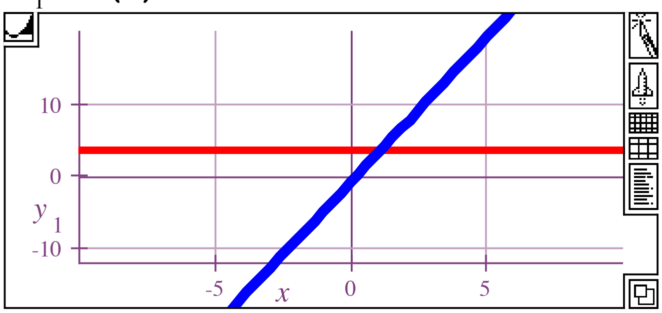
MR, 12/29: In this case, f '(a) is always greater than 0, and the minimum on this interval -10≤ x≤ 10 occurs where x=-10. In this case, yes, it is possible for f(a)≤f(x) (in this case f(-10)≤f(x) for all other x 's:

$f(x) = 4x$

$f'(x) = 4$

$y = f(x)$

$y_1 = f'(x)$



DD: 1/1/13: You are not on the right track with this problem. I 've uploaded a movie that I hope will help your understanding.

G.6.c.ii)

You 've got a function f(x) and a point x = a.

If f ′(a)<0, is it possible that f(a)≤f(x) for all other x 's? Why?

MR, 1/6: Since f '(a) is negative, when x moves to the right of a, then f(x) will decrease and when x moves to the left of a, then f(x) will increase. It is not possible for f(a)≤f(x) for all other x 's. If this were the case, f(x) would be at its global minimum and f '(a) would = 0.

DD: 1/9/13: Good.

MR, 1/2: Thanks for the video. Since f '(a) is negative, when f(x) moves to the right it will negative, when f(x) moves to the right it will decrease and when it moves to the left it will increase. It is not possible for $f(a) \leq f(x)$ for all other x 's If this were the case, f(x) would be at its global minimum and f '(a) would = 0 global minimum and f '(a) would = 0.

DD: 1/6/13: Ditto.

MR, 12/29: In this case, f '(a) is always less than 0, and the minimum on this interval -10≤ x≤ 10 occurs where x=10. In this case, yes, it is possible for $f(a) \leq f(x)$ (in this case f(10)≤f(x) for all other x 's.