Growth

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2.06 More Tools and Measurements:Techniques for Calculating Integrals

Give It a Try - G4

Graphics Primitives

DESolvers

This LiveMath Independence Declaration allows the derivatives to be computed as intended in this notebook.

The variables $(a, b, c, x, y, t, r, k, s, z, h, B)$ are independent of each other.

G.4) Tables of integrals via iteration

G.4.a.i)

Use the integration by parts formula to calculate

$$\int_1^e \ln(x)\,dx$$

by setting $u(x) = \ln(x)$ and $v'(x) = 1$.

$u(x) = \ln(x)$ and $v'(x) = 1$.

This gives

$u'(x) = \frac{1}{x}$ and $v(x) = x$.

Plug into the integration by parts formula to get

$$\int_1^e \ln(x)1\,dx$$

$$= \int_1^e u(x)v'(x)\,dx$$

$$= u(x)v(x)\Big|_1^e - \int_1^e v(x)u'(x)\,dx$$

$$= \ln(x)x\Big|_1^e - \int_1^e x\left(\frac{1}{x}\right)dx$$

$$= \ln(x)x \,\Big|_1^e - \int_1^e 1\,dx$$

$$= \ln(x)x \,\Big|_1^e - x\,\Big|_1^e$$

$$= (e\ln(e)-0)-(e-1)$$

$$= e\ln(e)-e+1$$

$$= e-e+1$$

$$=1$$

$$\int_1^e \ln(x)\,dx$$

$$\int_1^e \ln(x)\,dx = \left(\Big|_{x=1}^{x=e} \left[\ln[x]\left[\int dx\right]\right]\right) - \left(\int_1^e \left[\int dx\right]\left[\frac{1}{x}\,dx\right]\right)$$ *Integrate by*

$$\int_1^e \ln(x)\,dx = 1$$ *Simplify*

RC: 01/25/13: Good

G.4.a.ii)

Calculate an expression for

$$\int_1^e \ln(x)^k dx \text{ in terms of } \int_1^e \ln(x)^{k-1} dx$$

and use this expression and the result from part a.i) above to build a table of the value

$$\int_1^e \ln(x)^k dx \text{ for } k = 1,2,3,\ldots,15$$

evaluating only one actual integral.

Tip:

Use the integration by parts formula to get the iteration formula

$$\int_1^e \ln(x)^k dx = e - k\int_1^e \ln(x)^{k-1} dx.$$

You supply the details.

$u(x) = \ln(x)^k$ and $v'(x) = dx$.

This gives

$u'(x) = \dfrac{k\ln(x)^{k-1}}{x}$ and $v(x) = x$.

Plug into the integration by parts formula to get

$$\int_1^e \ln(x)^k dx$$

$$= \int_1^e u(x)v'(x)dx$$

$$= u(x)v(x) \Big|_1^e - \int_1^e v(x)u'(x)dx$$

$$= \ln(x)^k x \Big|_1^e - \int_1^e x\left(k\frac{\ln(x)^{k-1}}{x}\right)dx$$

$$= \ln(x)^k x \Big|_1^e - k\int_1^e \left(\ln(x)^{k-1}\right)dx$$

$$= \ln(x)x \Big|_1^e - k\int_1^e \left(\ln(x)^{k-1}\right)dx$$

$$= (e\ln(e) - 0) - (e - 1)$$

$$= e\ln(e) - e + 1$$

$$= e-k(\ln(x^{k-1})$$

$\text{Int}(m) = \int_1^e (\ln[m])^x\, dx$

$\text{Int}(1)$

$\triangle$ $\text{Int}(1) = \int_1^e (\ln[1])^x\, dx$ *Substitute*

$\triangle$ $1 = 0$ *Calculate*

$\text{Int}(2)$

$\triangle$ $\text{Int}(2) = \int_1^e (\ln[2])^x\, dx$ *Substitute*

$\triangle$ $2 = 0.883730584940656$ *Calculate*

$\text{Int}(e)$

$\triangle$ $\text{Int}(e) = \text{Int}(e)$ *Substitute*

$\triangle$ $\text{Int}(e) = 2$ *Simplify*

RC: 01/25/13: Now this needs to be set up as a recursion formula. See attached movie.

$f(x) = \int_0^1 (\ln[x])^{k-1}\, dx$

$\text{Int}(x) = \int_1^e (\ln[x])^1\, dx$

$\text{Int}(x) = 1$ *Simplify*

$\text{Int}(k) = \ln\left(\int_1^e x^{k-1}\, dx\right)$

$\text{Int}(k) = \ln(\text{Int}[k-1])$

$\text{Int}(k) = \begin{cases} \ln(\text{Int}[k-1]) & (k > 0) \\ 1 & (k = 1) \end{cases}$

$\text{Int}(2)$

$\text{Int}(2) = \begin{cases} \ln(\text{Int}[2-1]) & (2 > 0) \\ 1 & (2 = 1) \end{cases}$ *Substitute*

$\text{Int}(2) = \text{Int}(2)$ *Substitute*

$\text{Int}(2) = 2$ *Simplify*

$\text{Int}(3)$

$\text{Int}(3) = \begin{cases} \ln(\text{Int}[3-1]) & (3 > 0) \\ 1 & (3 = 1) \end{cases}$ *Substitute*

$\text{Int}(3) = \begin{cases} \text{Int}(3) & (3 > 0) \\ 1 & (3 = 1) \end{cases}$ *Substitute*

$3 = 3$ *Simplify*

RDC 1.26.13 I think this is what you were looking for.

RC: 02/04/13:Incorrect. Hint:

$\text{MyInt}(n) = \int_1^e (\ln[x])^n\, dx$

RC: 02/04/13:Starting recursion relation using your integral computation above:

$$\int_1^e \ln(x)^k dx = \ln(x)x \,\Big|_1^e - k\int_1^e \left(\ln(x)^{k-1}\right) dx$$

$\text{MyInt}(k) = \ln(x)x \,\Big|_1^e - k * \text{MyInt}(k-1)$

$\text{MyInt}(1)$

$\text{MyInt}(1) = \int_1^e (\ln[x])^1\, dx$ *Substitute*

$\text{MyInt}(1) = \left[\int \ln(x)\, dx\right]_{x=1}^{x=e}$ *Simplify*

$\triangle$ $\text{MyInt}(1) = \Big[x \ln(x) - x \Big]_{x=1}^{x=e}$ *Simplify*

$\triangle$ $\text{MyInt}(1) = (e \ln[e] - e) - (1 \ln[1] - 1)$ *Simplify*

$\triangle$ $\text{MyInt}(1) = 1$ *Simplify*

RC: 02/04/13: Describe how this is the recursion relation we seek:

RDC 2.4.13 You 're still blowing my mind on this one. The intent of the recursive formula is to use one or more preceding terms to predict the next term. When you state below e-n for MyInt(n-1) that means your n value is looking back when you search for 3. It is evident by the notation. It appears with MyInt as your functon you keep adjusting your original integral e to 1 based on the delta between 1 and the new value you seek.

$$\text{MyInt}(n) = \begin{cases} e - n\,\text{MyInt}(n-1) & (n > 1) \\ 1 & (n = 1) \end{cases}$$

RC: 02/04/13: Checking for n=3:

$\text{MyInt}(3)$

$\triangle$ $\text{MyInt}(3) = \begin{cases} e - 3\,\text{MyInt}(3-1) & (3 > 1) \\ 1 & (3 = 1) \end{cases}$ *Substitute*

$\triangle$ $\text{MyInt}(3) = -3\,\text{MyInt}(2) + e$ *Simplify*

$\triangle$ $\text{MyInt}(3) = -3\left(\begin{cases} e - 2\,\text{MyInt}[2-1] & [2 > 1] \\ 1 & [2 = 1] \end{cases}\right) + e$ *Substitute*

$\triangle$ $\text{MyInt}(3) = -3(-2\,\text{MyInt}[1] + e) + e$ *Simplify*

$\triangle$ $\text{MyInt}(3) = -3\left(-2\left[\begin{cases} e - 1\,\text{MyInt}\{1-1\} & \{1 > 1\} \\ 1 & \{1 = 1\} \end{cases}\right] + e\right) + e$ *Si*

$\triangle$ $\text{MyInt}(3) = -3(-2 \cdot 1 + e) + e$ *Simplify*

$\triangle$ $\text{MyInt}(3) = -3(e - 2) + e$ *Simplify*

$\triangle$ $\text{MyInt}(3) = -(-6 + 3e) + e$ *Expand*

$\triangle$ $\text{MyInt}(3) = -2e + 6$ *Simplify*

$\triangle$ $\text{MyInt}(3) = 0.56343634308191$ *Calculate*

$\int_1^e (\ln[x])^3 \, dx$

$\triangle$ $\int_1^e (\ln[x])^3 \, dx = 0.563436342134966$ *Calculate*

G.4.b)

Use integration by parts to prepare a table of the values of

$$\int_0^\pi \cos(x)^n dx$$

for $n = 1,2,3,...,20$ by evaluating only two actual integrals.

Here 's a little trick:

$$\int_0^{\pi} \cos(x)^n dx = \int_0^{\pi} \text{-}\sin(x)^{n-1}\cos(x) dx.$$

The integration by parts formula is

$$\int_0^{\pi} u(x)v'(x)dx = u(x)v(x) \Big|_0^{\pi} - \int_0^{\pi} v(x)u'(x)dx$$

Make the assignments:

$u(x) = \cos(x)^{n-1}$ and $v'(x) = \cos(x)$.

This gives

$u'(x) = \text{-}(n-1)\cos(x)^{n-2}\sin(x)$ and $v(x) = \sin(x)$.

With these assignments,

$$\int_0^{\pi} \cos(x)^n dx = \int_0^{\pi} \text{-}\sin(x)^{n-1}\cos(x) dx$$

$$= \int_0^{\pi} u(x)v'(x)dx$$

$$= u(x)v(x) \Big|_0^{\pi} - \int_0^{\pi} v(x)u'(x)dx$$

$$= \cos(x)^{n-1}\sin(x) \Big|_0^{\pi}$$

$$- \int_0^{\pi} \sin(x)(\text{-}(n-1)\cos(x)^{n-2}\sin(x))dx$$

$$= 0\text{-}((n-1)\int_0^{\pi} \cos(x)^{n-2}\sin(x)^2 dx)$$

$$= \text{-}n+1\int_0^{\pi} \cos(x)^{n-2}((1\text{-}\cos(x)^2)dx$$

$$=(n-1)\int_0^{\pi} \cos(x)^{n-2}dx\text{-}(n-1)\int_0^{\pi} \cos(x)^n dx$$

The upshot:

$$\int_0^{\pi} \cos(x)^n dx$$

$$=-(n-1)\int_0^{\pi} \cos(x)^{n-2}dx+(n-1)\int_0^{\pi} \cos(x)^n dx.$$

This looks bad, but it feels good because when you put

$$\text{Int}(n) = \int_0^{\pi} \cos(x)^n dx,$$

then you can see that:

RC: 02/07/13: Now do this problem correctly, using the previous problem as a model.

$\text{Int}(n) = -(n+1)\,\text{Int}(n-2) - (n-1)\,\text{Int}(n)$

$\text{Int}(n) + (n-1)\,\text{Int}(n) = -(n+1)\,\text{Int}(n-2)$ *Move Over*

$n\,\text{Int}(n) = -(n+1)\,\text{Int}(n-2)$ *Collect*

$\text{Int}(n) = -\dfrac{(n+1)\,\text{Int}(n-2)}{n}$ *Isolate*

$\text{Int}(n) = \int_0^{\pi} (\cos[x])^{n}\, dx$

Reduction formula:

$\text{Int}(n) = -\dfrac{(n-1)\,\text{Int}(n-2)}{n}$

Examples

Int (1)

$\text{Int}(1) = \int_0^{\pi} (\cos[x])^1\, dx$ *Substitute*

$\text{Int}(1) = \Big[\int \cos(x)\, dx\Big]_{x=0}^{x=\pi}$ *Simplify*

$\text{Int}(1) = \sin(\pi) - \sin(0)$ *Simplify*

$\text{Int}(1) = 1$

Int (2)

$\text{Int}(2) = \int_0^{\pi} (\cos[x])^2\, dx$ *Substitute*

With Reduction Formula first

$\text{Int}(2) = -\dfrac{(2-1)\,\text{Int}(2-2)}{2}$ *Substitute*

$\text{Int}(2) = -\dfrac{(2-1)\,\text{Int}(0)}{2}$ *Simplify*

$$\triangle\ \text{Int}(2) = -\frac{(2-1)\left(\int_0^{\pi} [\cos\{x\}]^0\, dx\right)}{2}$$ *Substitute*

$$\triangle\ \text{Int}(2) = -\tfrac{1}{2}(1\,\pi - 1\cdot 0)$$ *Simplify*

$$\triangle\ \text{Int}(2) = -\tfrac{1}{2}\,\pi$$ *Simplify*

$$\triangle\ \text{Int}(2) = -\tfrac{1}{2}\,\pi$$ *Simplify*

☐ Int(3)

$$\triangle\ \text{Int}(3) = \int_0^{\pi} (\cos[x])^3\, dx$$ *Substitute*

With Reduction Formula first

$$\triangle\ \text{Int}(3) = -\frac{(3-1)\,\text{Int}(3-2)}{3}$$ *Substitute*

$$\triangle\ \text{Int}(3) = -\frac{(3-1)\,\text{Int}(1)}{3}$$ *Simplify*

Substitute from previous computation of Int(1)

$$\triangle\ \text{Int}(3) = -\frac{(3-1)\cdot 1}{3}$$ *Substitute*

$$\triangle\ \text{Int}(3) = -\tfrac{2}{3}$$ *Simplify*