



Matrices, Geometry & *Mathematica*

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MGM.02 2D Matrix Action *TUTORIALS*

Mathematica Initializations

T.1) Hitting with a 2D matrix and visual assessment of the result

□ T.1.a.i) A hit with $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ stretches:

Go with

$$A = \begin{pmatrix} 2.0 & 0 \\ 0 & 3.0 \end{pmatrix}$$

and use an action movie to analyze the action resulting from hitting the unit circle with A.

□ Answer:

Hit it!

```
A =  $\begin{pmatrix} 2.0 & 0 \\ 0 & 3.0 \end{pmatrix}$ ;
Clear[x, y, t, hitplotter, hitpointplotter, pointcolor, actionarrows, matrix2D];
{tlow, thigh} = {0, 2 π};

{x[t_], y[t_]} = {Cos[t], Sin[t]};

pointcolor[t_] = RGBColor[0.5 (Cos[t] + 1), 0.5 (Sin[t] + 1), 0];

jump =  $\frac{\text{thigh} - \text{tlow}}{16}$ ;

hitplotter[matrix2D_] := ParametricPlot[matrix2D.{x[t], y[t]},
  {t, tlow, thigh}, PlotStyle → {{Thickness[0.01], NavyBlue}},
  PlotRange → {{-Max[{1.2, Max[SingularValues[A][[2]]]}],
    Max[{1.2, Max[SingularValues[A][[2]]]}]},
  {-Max[{1.2, Max[SingularValues[A][[2]]]}], Max[
```

```

    {1.2, Max[SingularValues[A][[2]]]}},
    AxesLabel → {"x", "y"}, DisplayFunction → Identity];

hitpointplotter[matrix2D_] :=
  Table[Graphics[{pointcolor[t], PointSize[0.035], Point[matrix2D.{x[t], y[t]}]}],
    {t, tlow, thigh - jump, jump}];

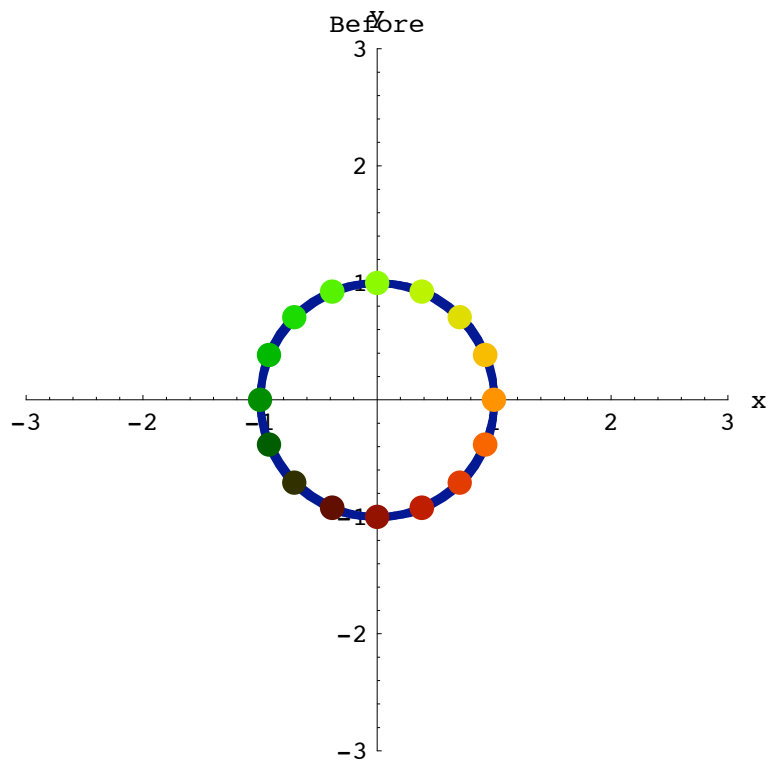
actionarrows[matrix2D_] :=
  Table[Arrow[matrix2D.{x[t], y[t]} - {x[t], y[t]}, Tail → {x[t], y[t]},
    VectorColor → pointcolor[t], HeadSize → 0.25], {t, tlow, thigh - jump, jump}];

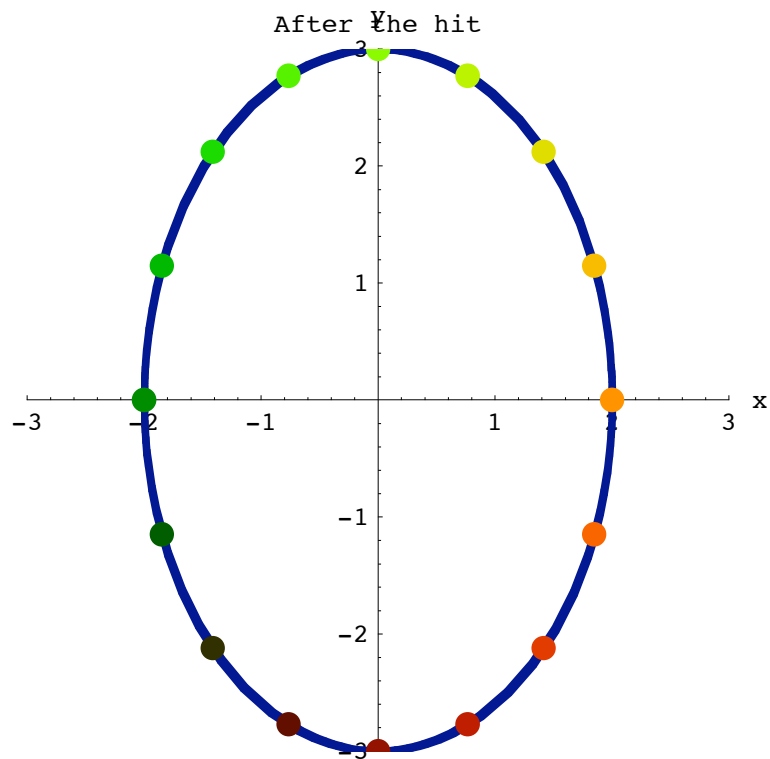
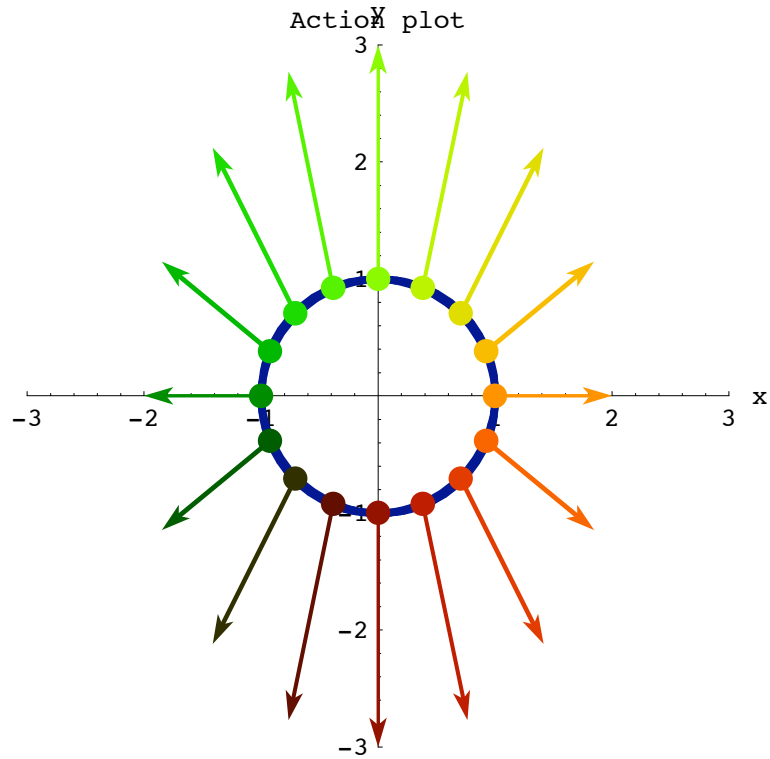
before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],
  PlotLabel → "Before", DisplayFunction → $DisplayFunction];

Show[before, actionarrows[A],
  PlotLabel → "Action plot", DisplayFunction → $DisplayFunction];

Show[hitplotter[A], hitpointplotter[A],
  PlotLabel → "After the hit", DisplayFunction → $DisplayFunction];

```





Grab all three plots, align and animate slowly.
 To get maximum viewing pleasure and maximum information,
 look at one point and follow it.
 Then pick another point and follow it.
 Keep doing this until you have a good visual grasp of the action.

This matrix is a pure stretcher along the x and y axes.

□ T.1.a.ii) A hit with $\begin{pmatrix} 0.5 & 0.0 \\ 0.0 & 0.7 \end{pmatrix}$ shrinks

Go with

$$A = \begin{pmatrix} 0.5 & 0.0 \\ 0.0 & 0.7 \end{pmatrix}$$

and use an action movie to analyze the action resulting from hitting the unit circle with A.

□ Answer:

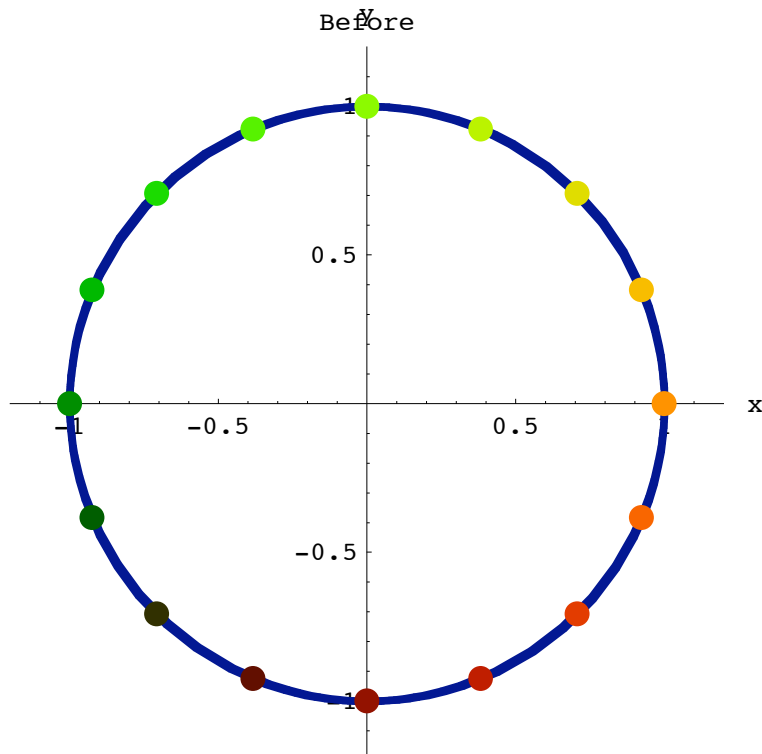
Hit it!

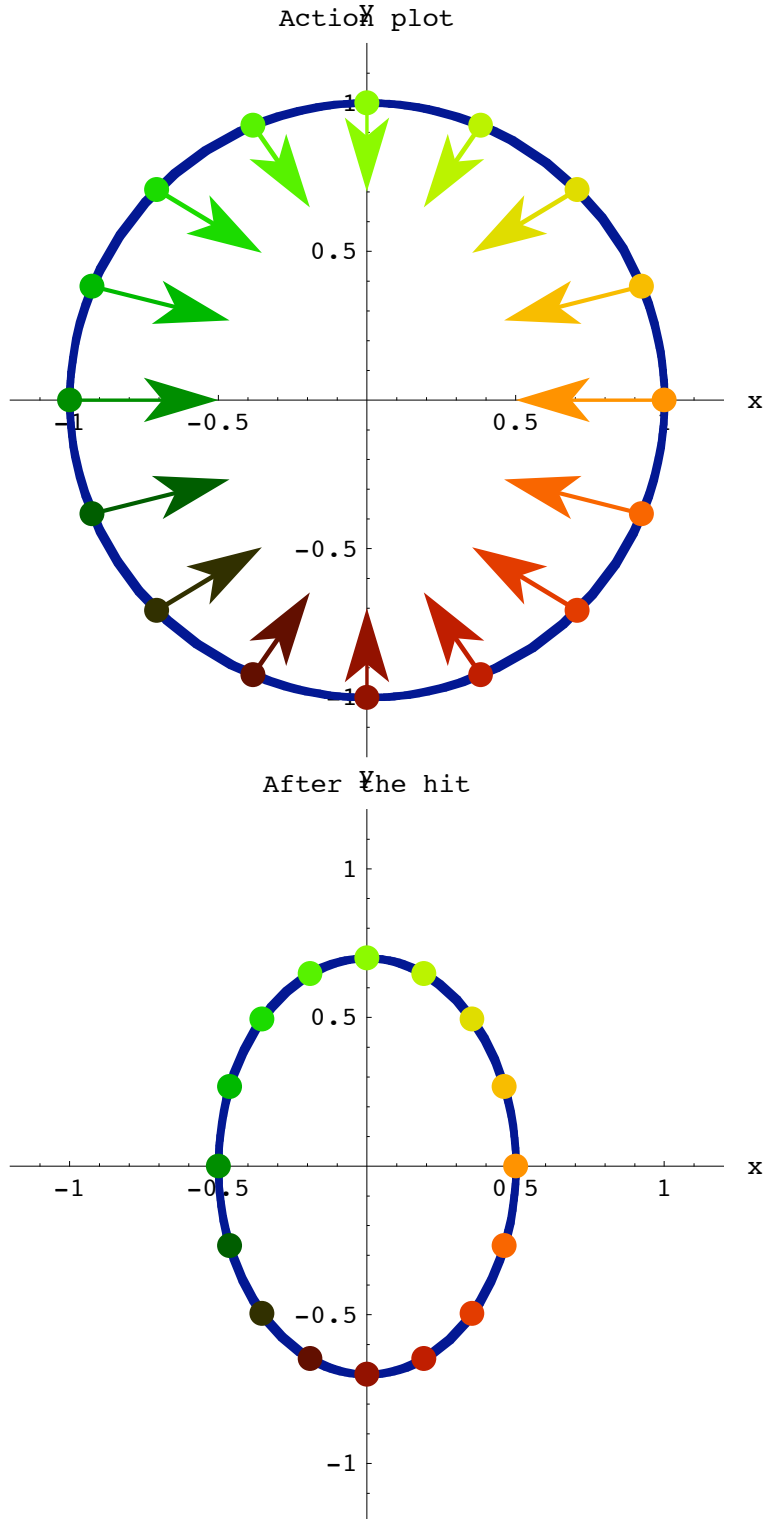
```
A = { 0.5 0.0 };
      { 0.0 0.7 };

before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],
  PlotLabel -> "Before", DisplayFunction -> $DisplayFunction];

Show[before, actionarrows[A],
  PlotLabel -> "Action plot", DisplayFunction -> $DisplayFunction];

Show[hitplotter[A], hitpointplotter[A],
  PlotLabel -> "After the hit", DisplayFunction -> $DisplayFunction];
```





Grab all three plots, align and animate slowly.
 To get maximum viewing pleasure and maximum information,
 look at one point and follow it.
 Then pick another point and follow it.
 Keep doing this until you have a good visual grasp of the action.

This matrix shrinks along the x and y axes.

□ T.1.a.iii) A hit with $\begin{pmatrix} 2.0 & 0.0 \\ 0.0 & 0.3 \end{pmatrix}$ stretches and shrinks

Go with

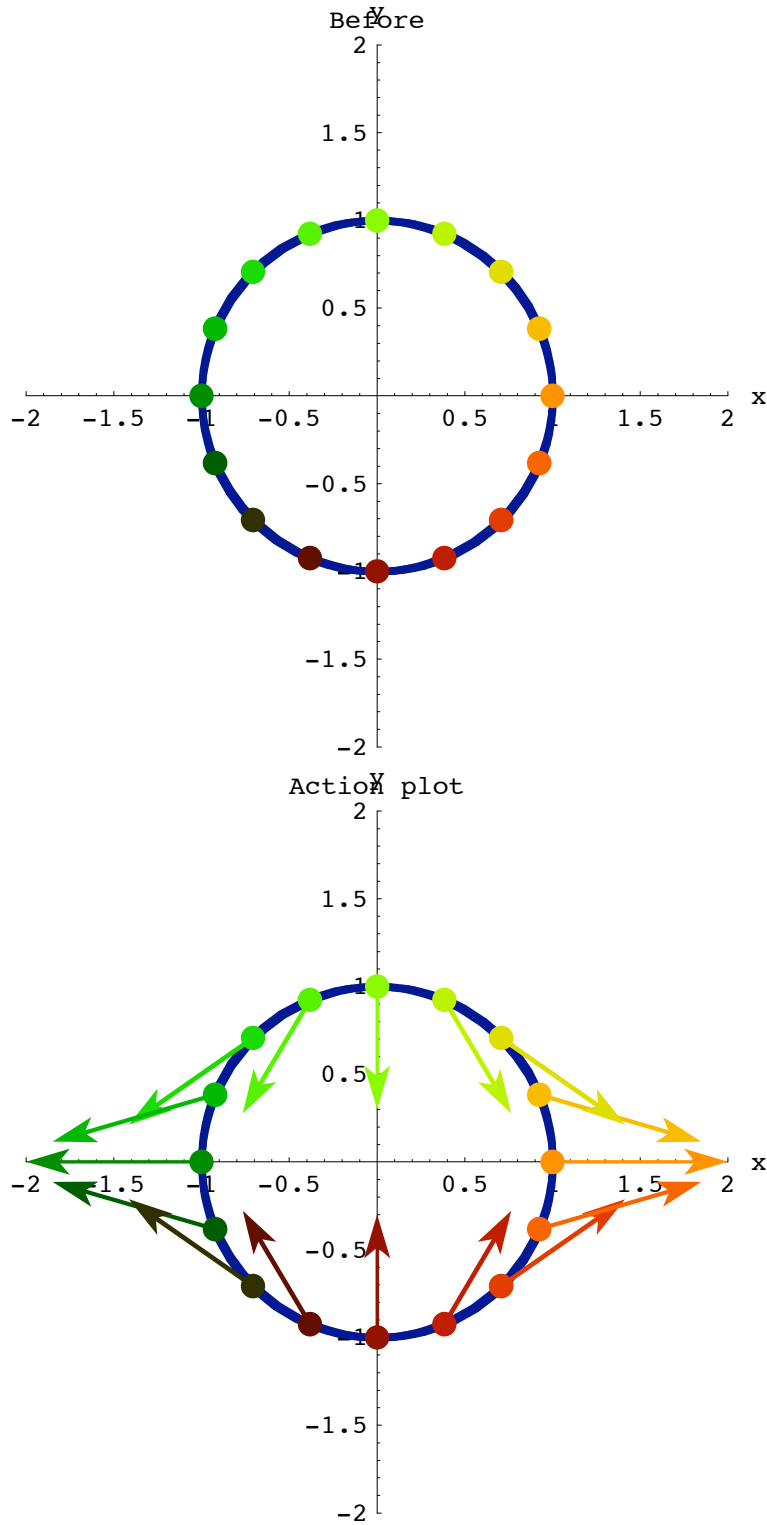
$$A = \begin{pmatrix} 2.0 & 0.0 \\ 0.0 & 0.3 \end{pmatrix}$$

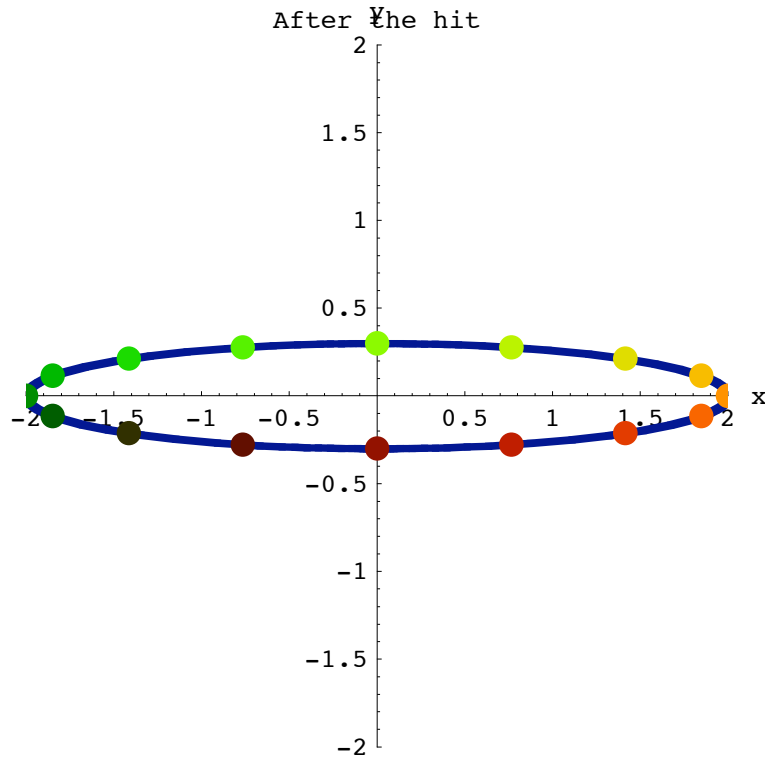
and use an action movie to analyze the action resulting from hitting the unit circle with A.

□ Answer:

Hit it!

```
A =  $\begin{pmatrix} 2.0 & 0.0 \\ 0.0 & 0.3 \end{pmatrix}$ ;  
ranger = 2;  
  
before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],  
  PlotLabel → "Before", DisplayFunction → $DisplayFunction];  
  
Show[before, actionarrows[A],  
  PlotLabel → "Action plot", DisplayFunction → $DisplayFunction];  
  
Show[hitplotter[A], hitpointplotter[A],  
  PlotLabel → "After the hit", DisplayFunction → $DisplayFunction];
```





Grab all three plots, align and animate slowly.
 To get maximum viewing pleasure and maximum information,
 look at one point and follow it.
 Then pick another point and follow it.
 Keep doing this until you have a good visual grasp of the action.

A hit with this matrix shrinks along the y-axis and stretches along the x-axis.

□ T.1.a.iv) A hit with $\begin{pmatrix} 1.18 & -0.56 \\ 1.61 & 0.41 \end{pmatrix}$ stretches, shrinks and rotates

Go with

$$A = \begin{pmatrix} 1.18 & -0.56 \\ 1.61 & 0.41 \end{pmatrix}$$

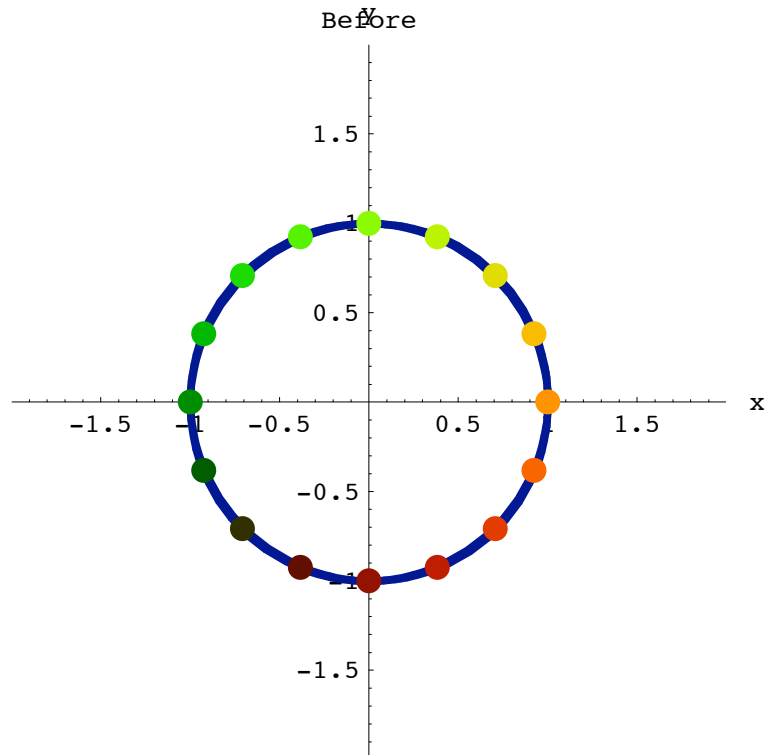
and use an action movie to analyze the action resulting from hitting the unit circle with A.

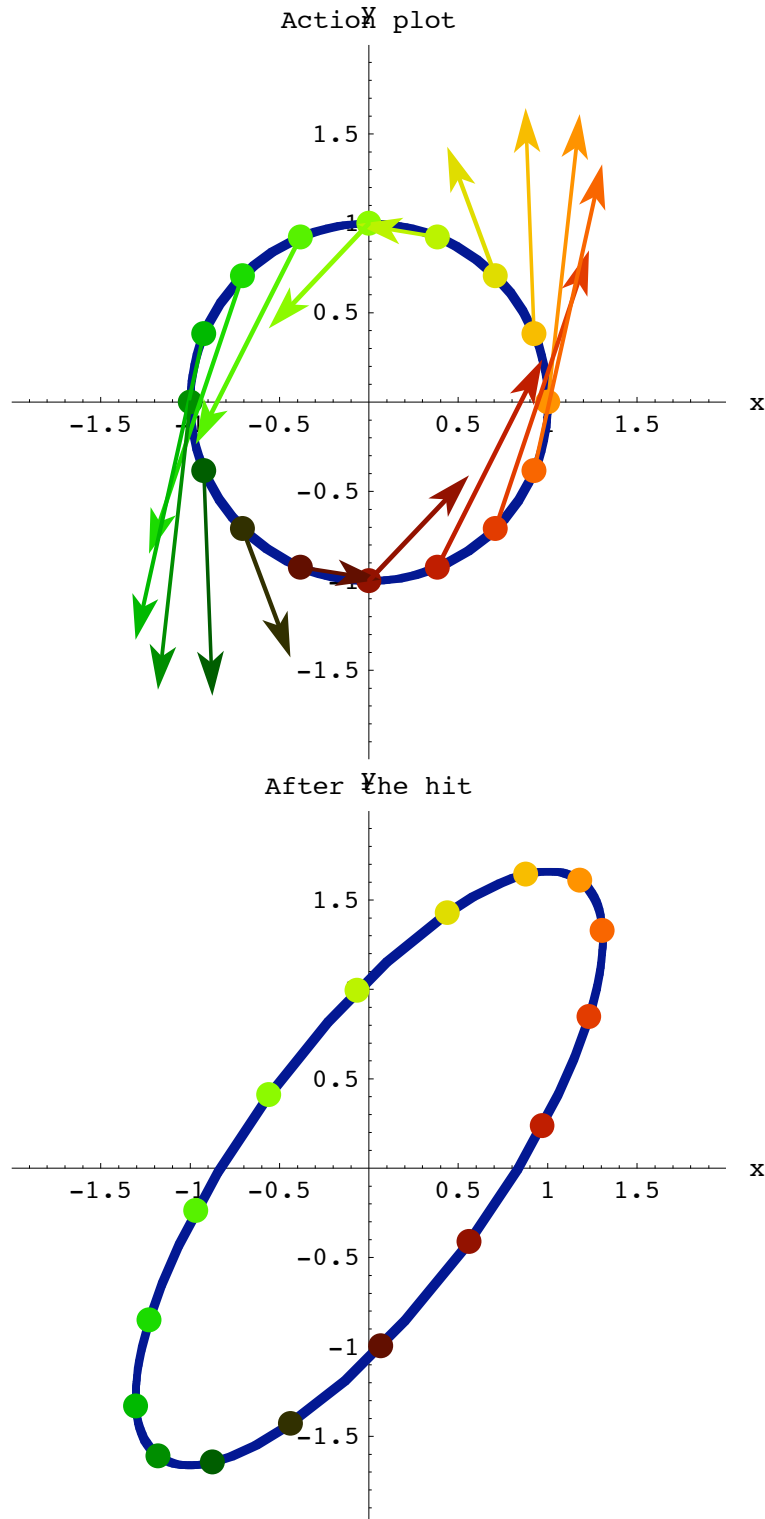
□ Answer:

Hit it!

$$A = \begin{pmatrix} 1.18 & -0.56 \\ 1.61 & 0.41 \end{pmatrix};$$


```
before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],  
  PlotLabel -> "Before", DisplayFunction -> $DisplayFunction];  
  
Show[before, actionarrows[A],  
  PlotLabel -> "Action plot", DisplayFunction -> $DisplayFunction];  
  
Show[hitplotter[A], hitpointplotter[A],  
  PlotLabel -> "After the hit", DisplayFunction -> $DisplayFunction];
```





Grab all three plots, align and animate slowly.
 To get maximum viewing pleasure and maximum information,
 look at one point and follow it.
 Then pick another point and follow it.
 Keep doing this until you have a good visual grasp of the action.

A hit with this matrix shrinks, stretches and rotates.

□ T.1.a.v) A hit with $\begin{pmatrix} 0 & 1.0 \\ 1.0 & 0 \end{pmatrix}$ flips

Go with

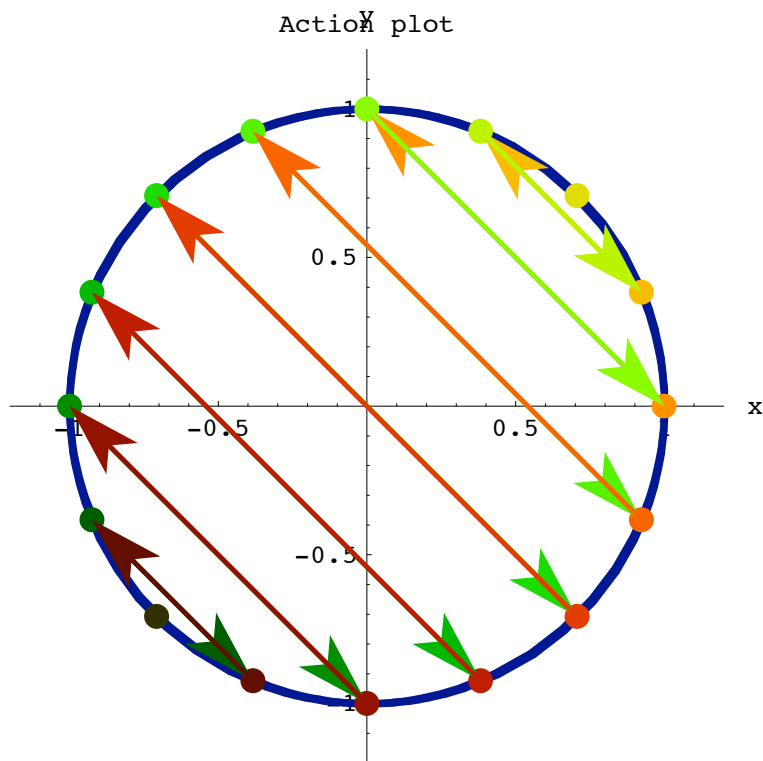
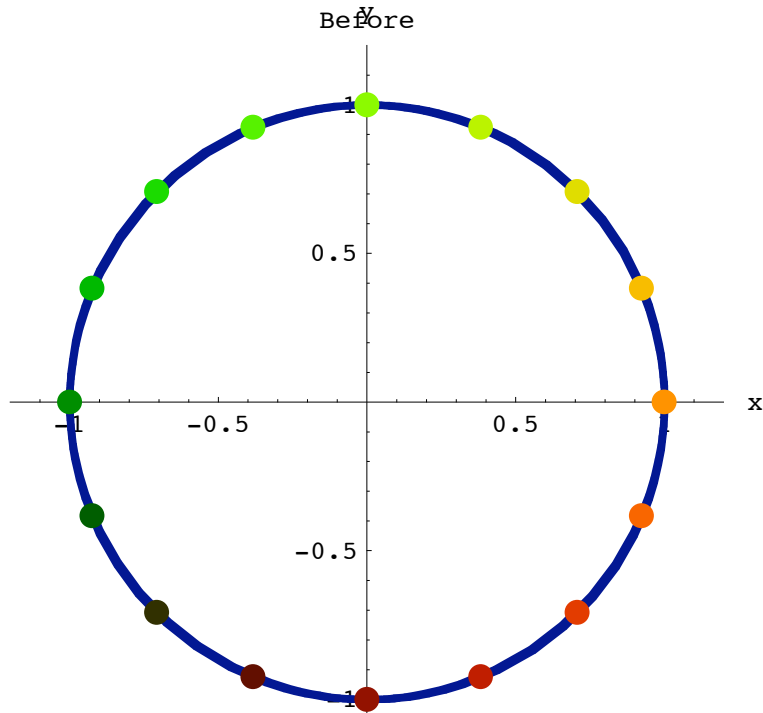
$$A = \begin{pmatrix} 0 & 1.0 \\ 1.0 & 0 \end{pmatrix}$$

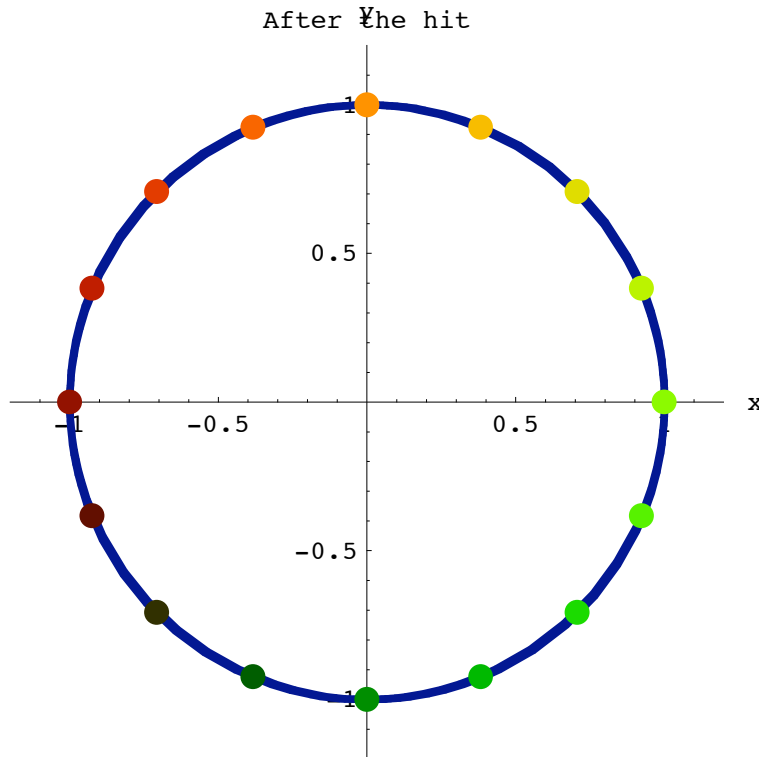
and use an action movie to analyze the action resulting from hitting the unit circle with A.

□ Answer:

Hit it!

```
A =  $\begin{pmatrix} 0 & 1.0 \\ 1.0 & 0 \end{pmatrix}$ ;  
ranger = 1.4;  
  
before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],  
  PlotLabel -> "Before", DisplayFunction -> $DisplayFunction];  
  
Show[before, actionarrows[A],  
  PlotLabel -> "Action plot", DisplayFunction -> $DisplayFunction];  
  
Show[hitplotter[A], hitpointplotter[A],  
  PlotLabel -> "After the hit", DisplayFunction -> $DisplayFunction];
```





Grab all three plots, align and animate slowly.
 To get maximum viewing pleasure and maximum information,
 look at one point and follow it.
 Then pick another point and follow it.
 Keep doing this until you have a good visual grasp of the action.

A hit with this matrix flips.

□ T.1.a.vi) A hit with $\begin{pmatrix} 0.35 & 1.65 \\ 1.65 & 0.35 \end{pmatrix}$ flips and stretches

Go with

$$A = \begin{pmatrix} 0.35 & 1.65 \\ 1.65 & 0.35 \end{pmatrix}$$

and use an action movie to analyze the action resulting from hitting the unit circle with A.

□ Answer:

Hit it!

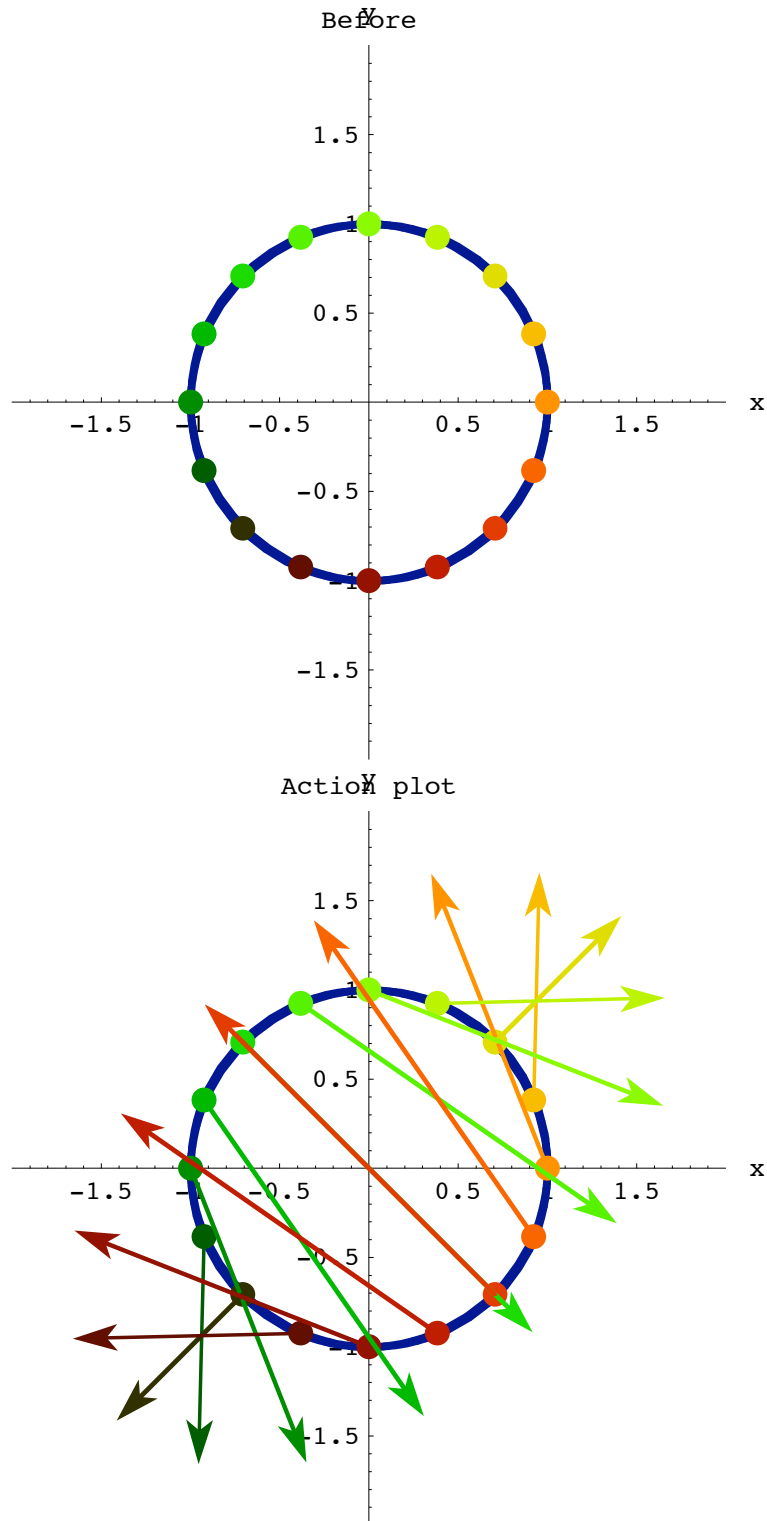
```

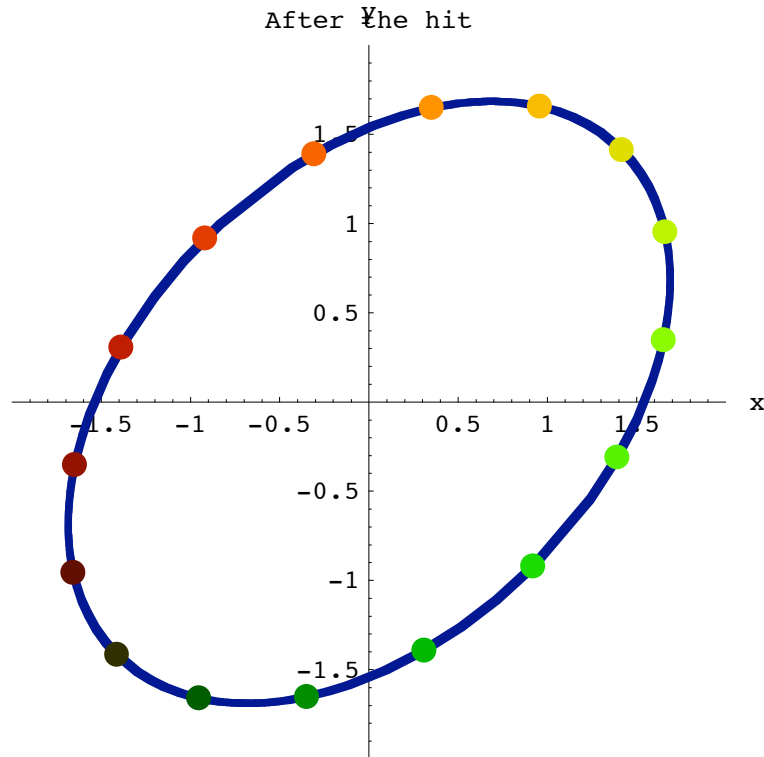
A = { 0.35 1.65 };
    { 1.65 0.35 };
ranger = 2;

before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],
  PlotLabel -> "Before", DisplayFunction -> $DisplayFunction];

```

```
Show[before, actionarrows[A],  
      PlotLabel -> "Action plot", DisplayFunction -> $DisplayFunction];  
  
Show[hitplotter[A], hitpointplotter[A],  
      PlotLabel -> "After the hit", DisplayFunction -> $DisplayFunction];
```





Grab all three plots, align and animate slowly.
 To get maximum viewing pleasure and maximum information,
 look at one point and follow it.
 Then pick another point and follow it.
 Keep doing this until you have a good visual grasp of the action.

A hit with this matrix stretches and flips.

□ T.1.a.vii) A hit with this $\begin{pmatrix} 1.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix}$ rotates, stretches and smashes

Go with

$$A = \begin{pmatrix} 1.2 & 0.8 \\ 0.6 & 0.4 \end{pmatrix}$$

and use an action movie to analyze the action resulting from hitting the unit circle with A.

□ Answer:

Hit it!

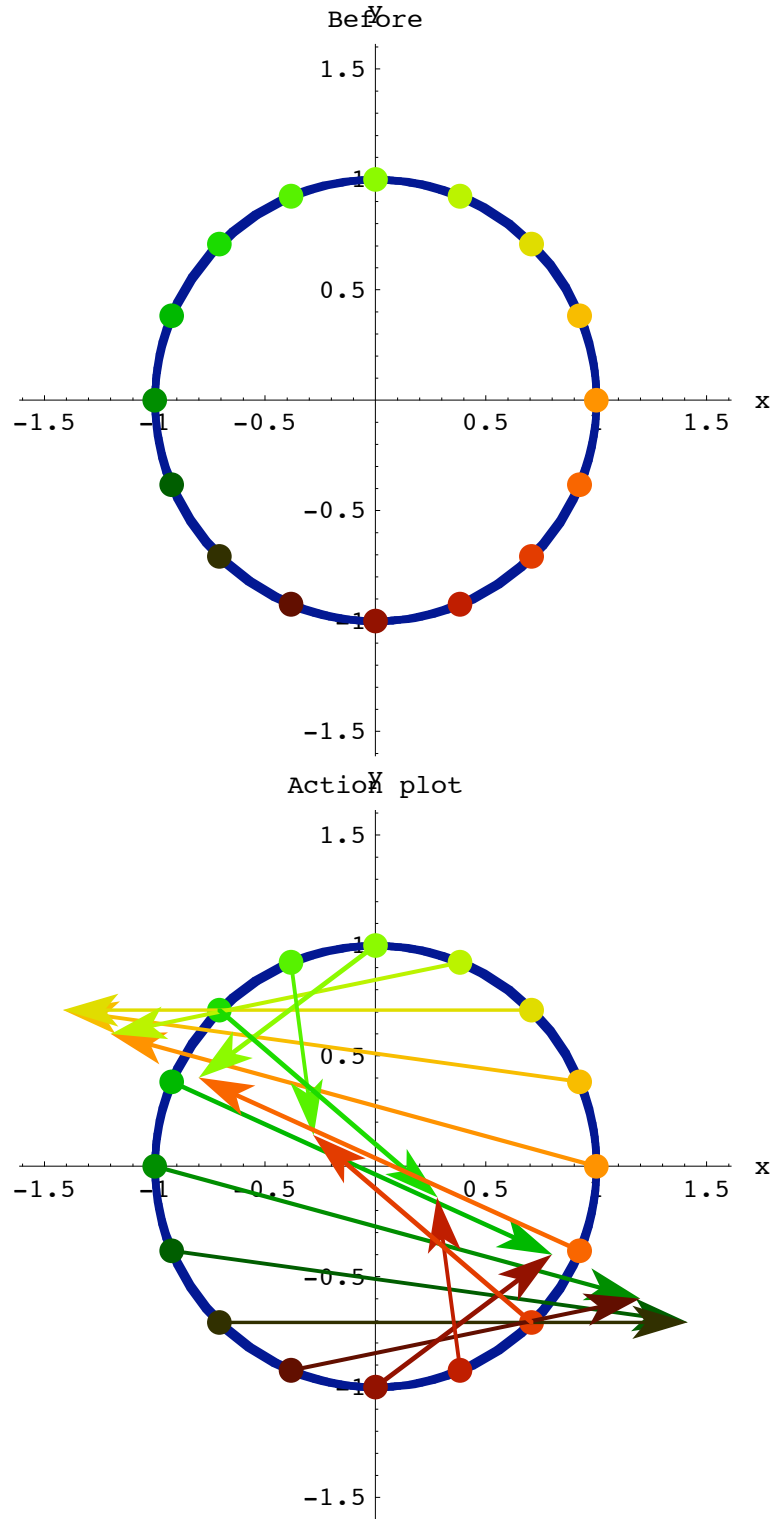
```

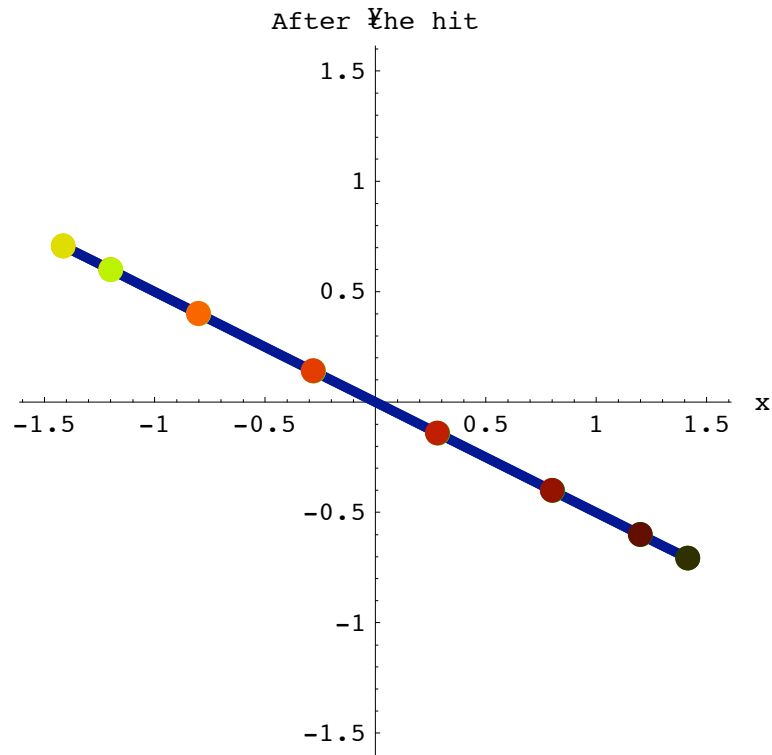
A = { -1.2  -0.8 };
    { 0.6   0.4 };
ranger = 2;

before = Show[hitplotter[IdentityMatrix[2]], hitpointplotter[IdentityMatrix[2]],
  PlotLabel -> "Before", DisplayFunction -> $DisplayFunction];

```

```
Show[before, actionarrows[A],  
      PlotLabel -> "Action plot", DisplayFunction -> $DisplayFunction];  
  
Show[hitplotter[A], hitpointplotter[A],  
      PlotLabel -> "After the hit", DisplayFunction -> $DisplayFunction];
```





Grab all three plots, align and animate slowly.
To get maximum viewing pleasure and maximum information,
look at one point and follow it.
Then pick another point and follow it.
Keep doing this until you have a good visual grasp of the action.

A hit with this matrix rotates, stretches and smashes.